

# Structured Electronic Design

## Estimation of poles and zeros in networks without feedback

# Time-constant matrix

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Basis for intuitive determination of poles in networks without feedback

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MNA matrix in first-order differential form:



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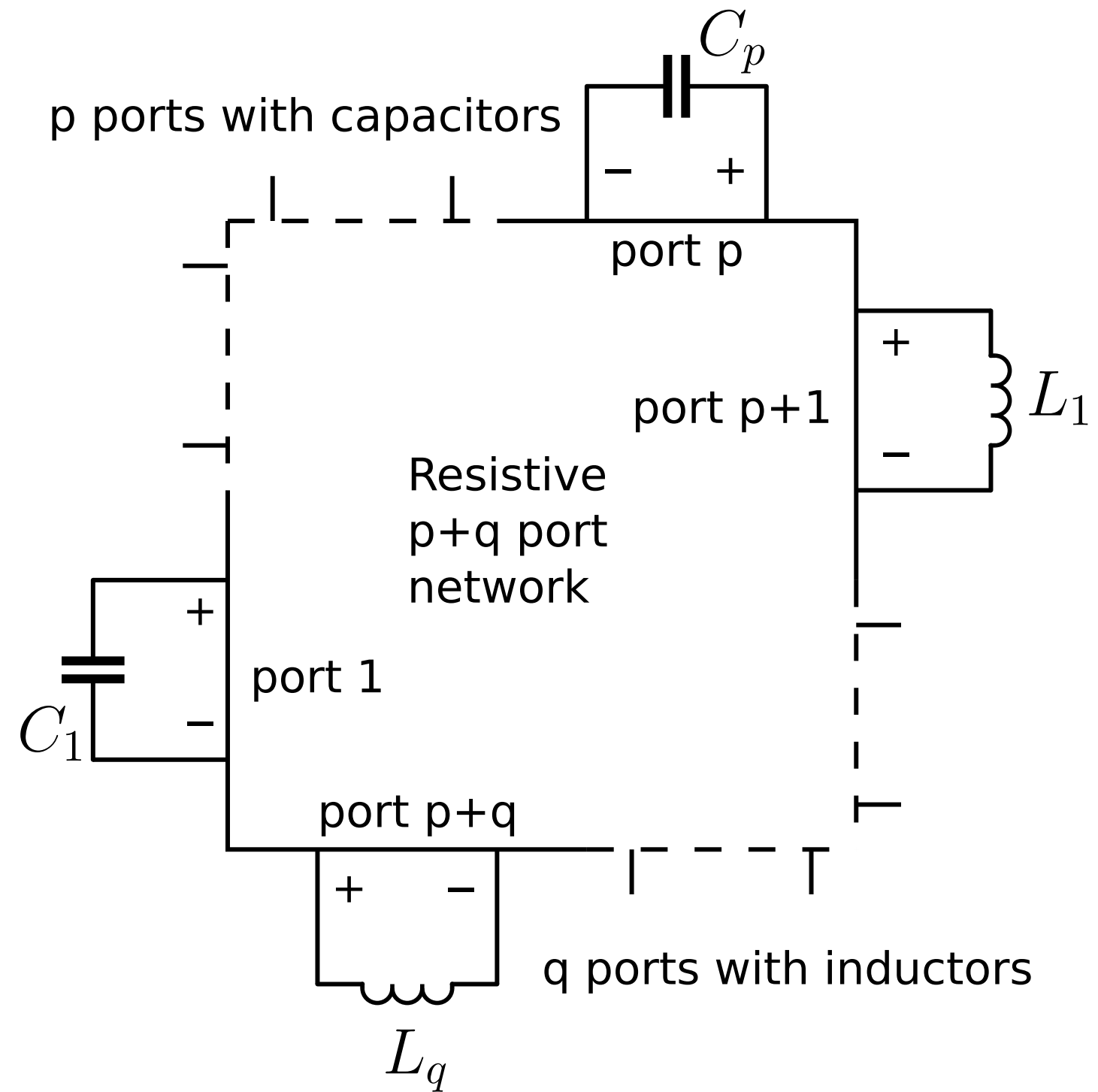
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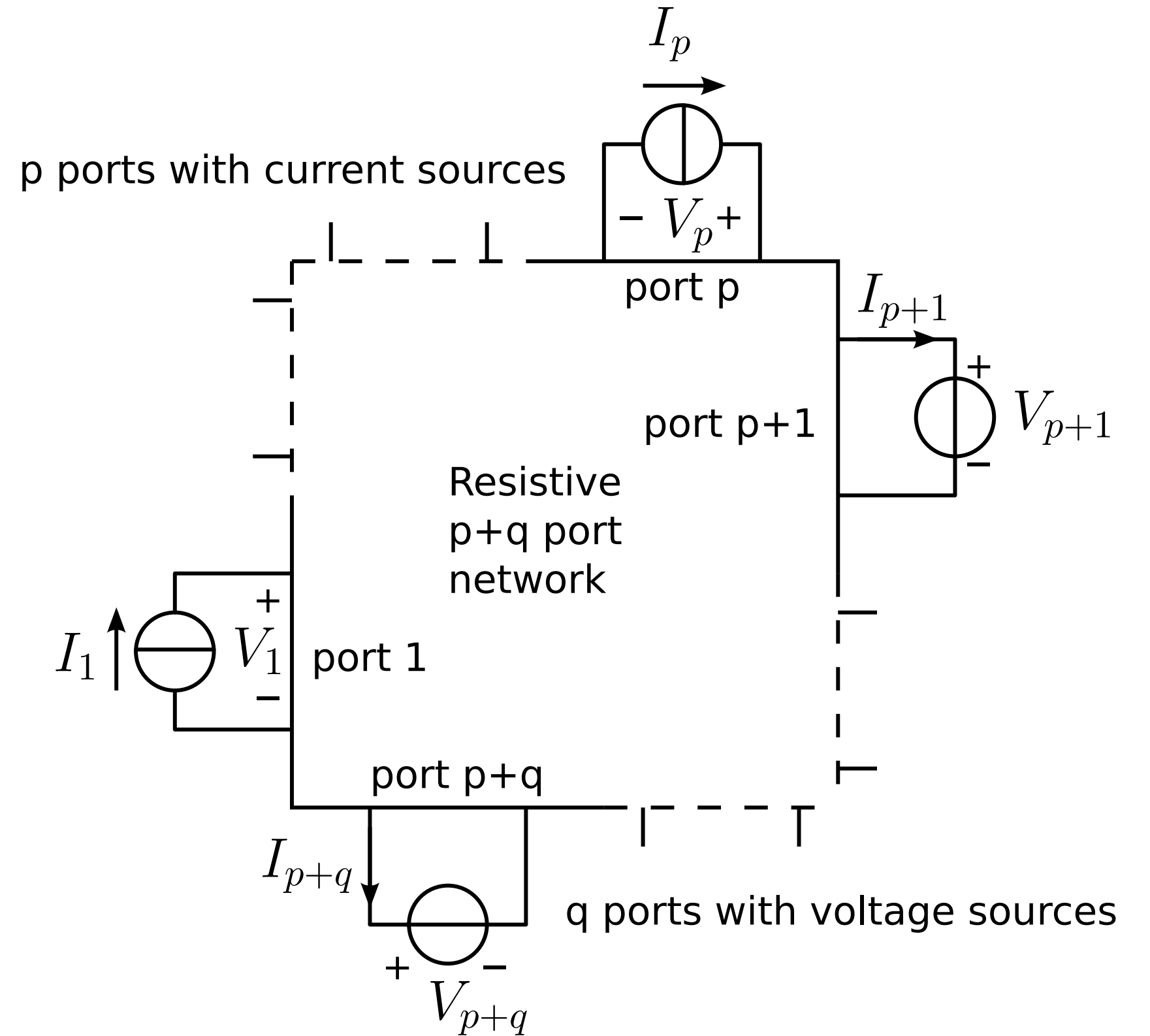
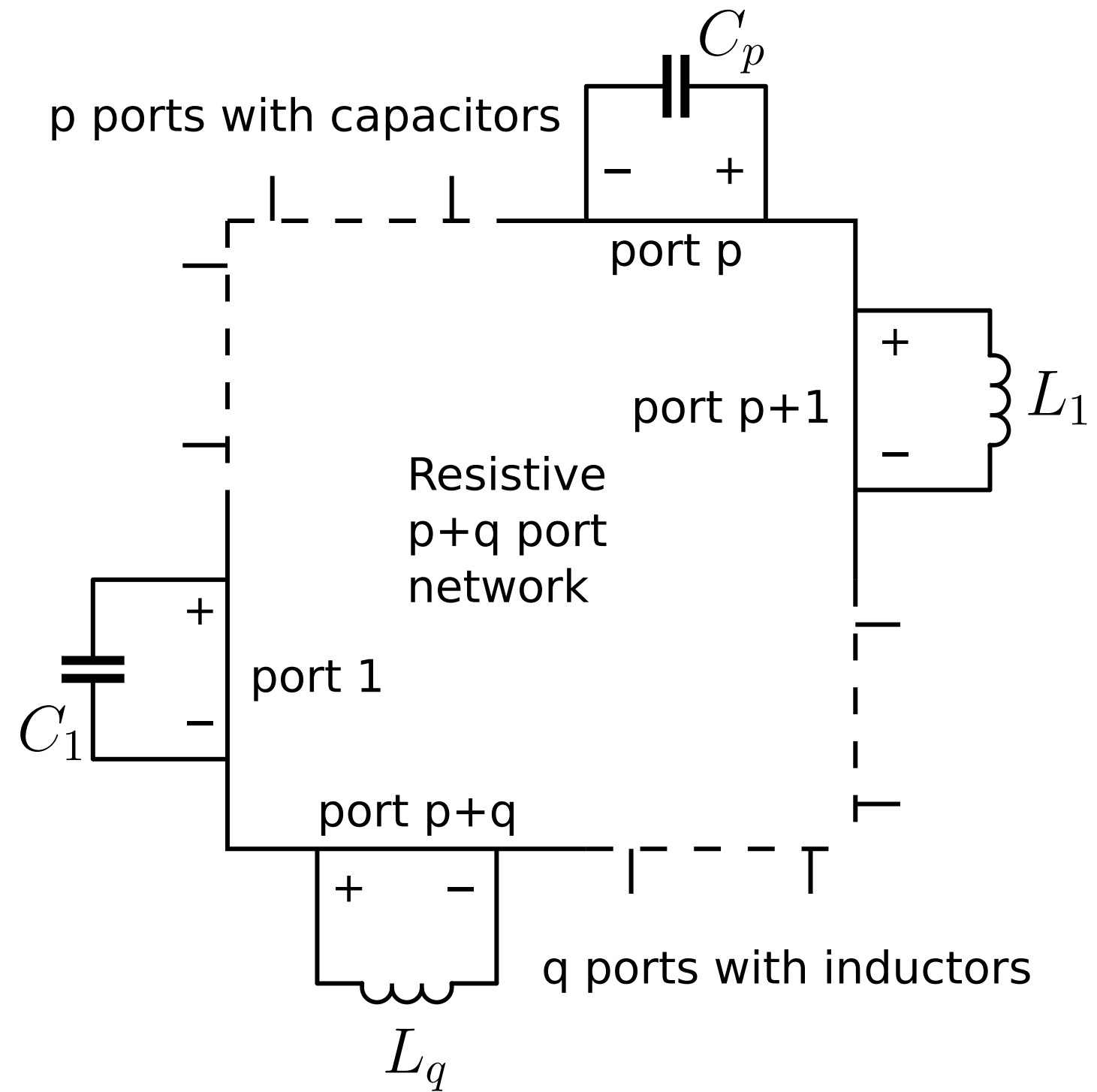
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# Resistance matrix

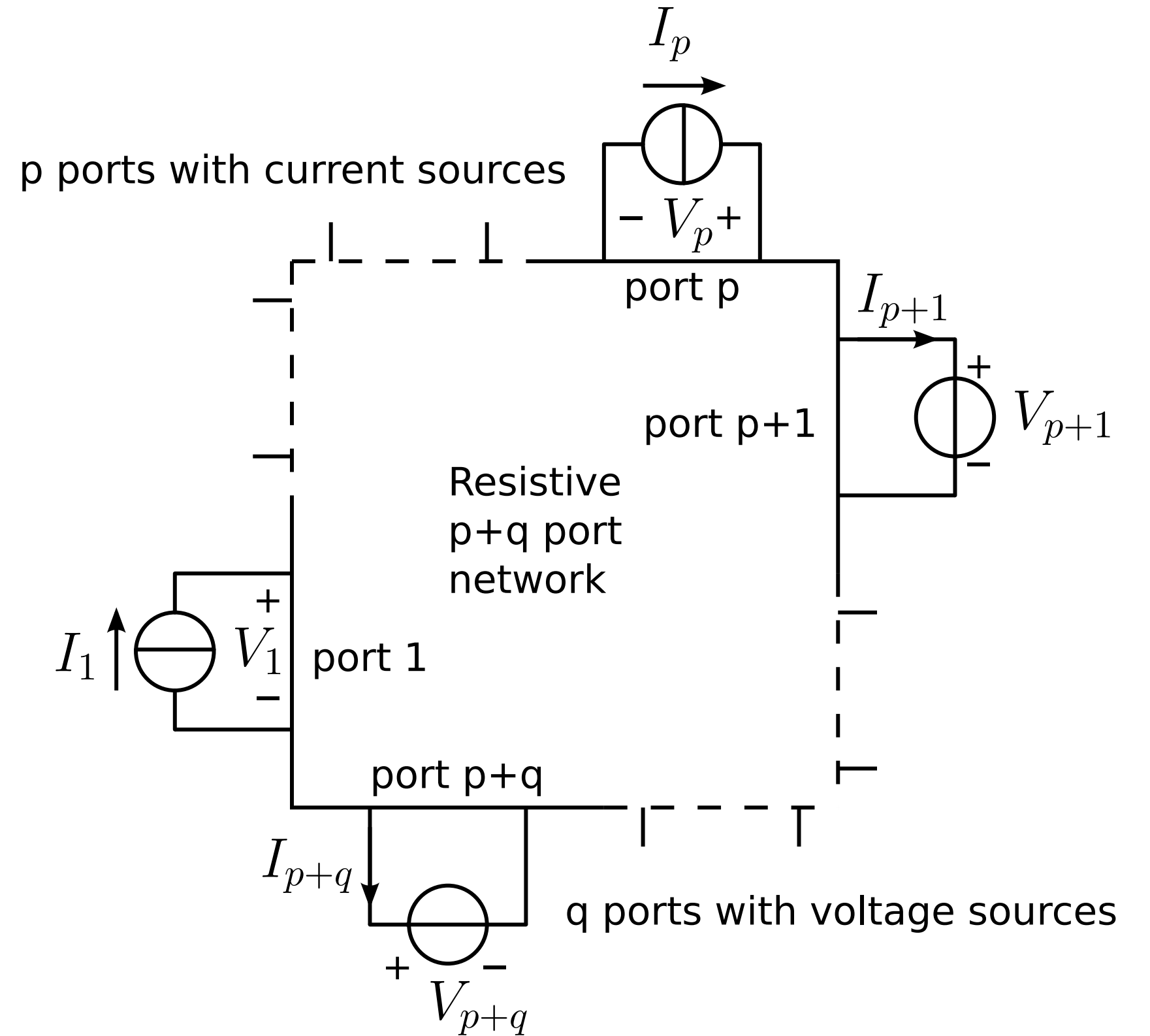
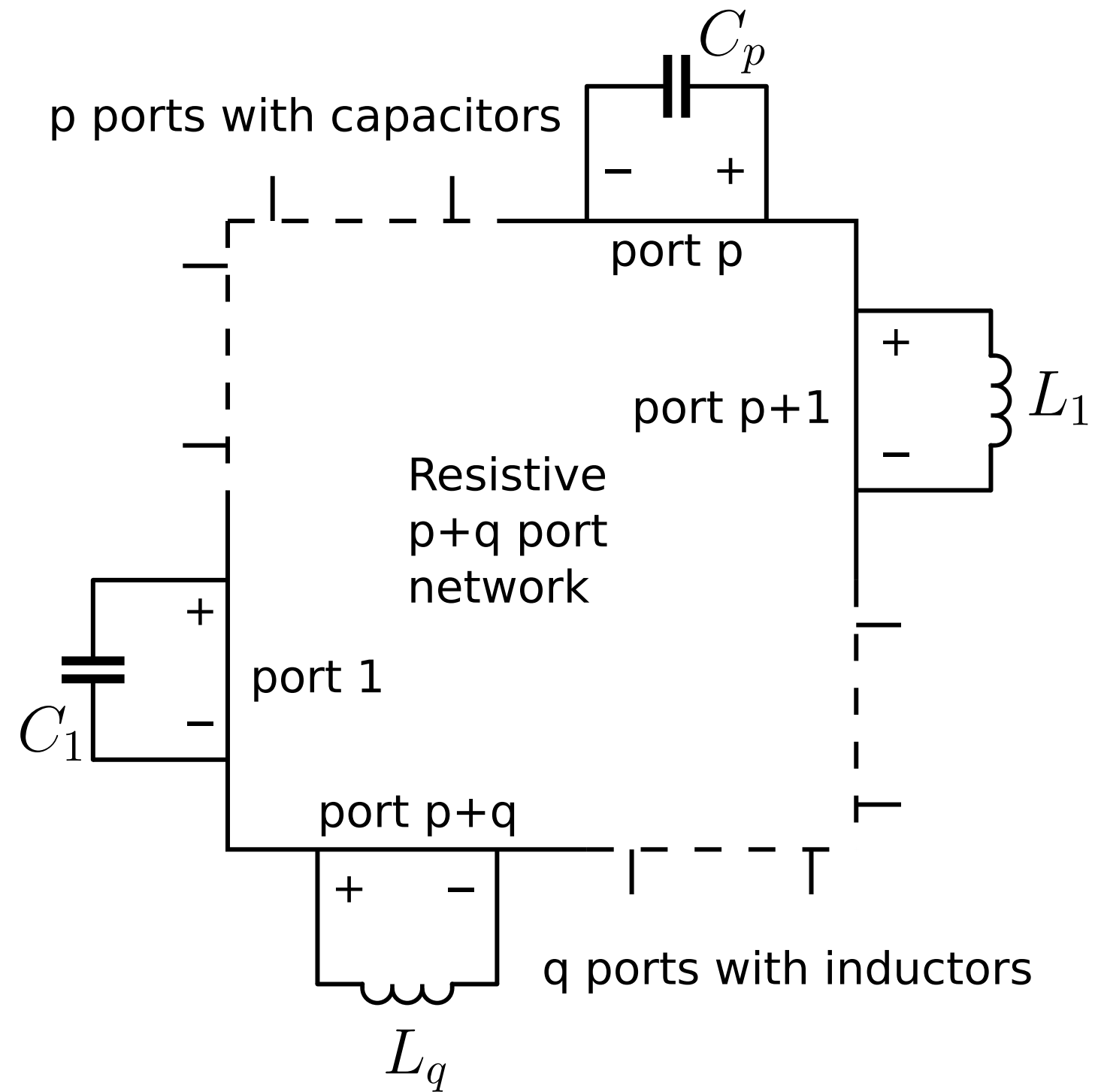
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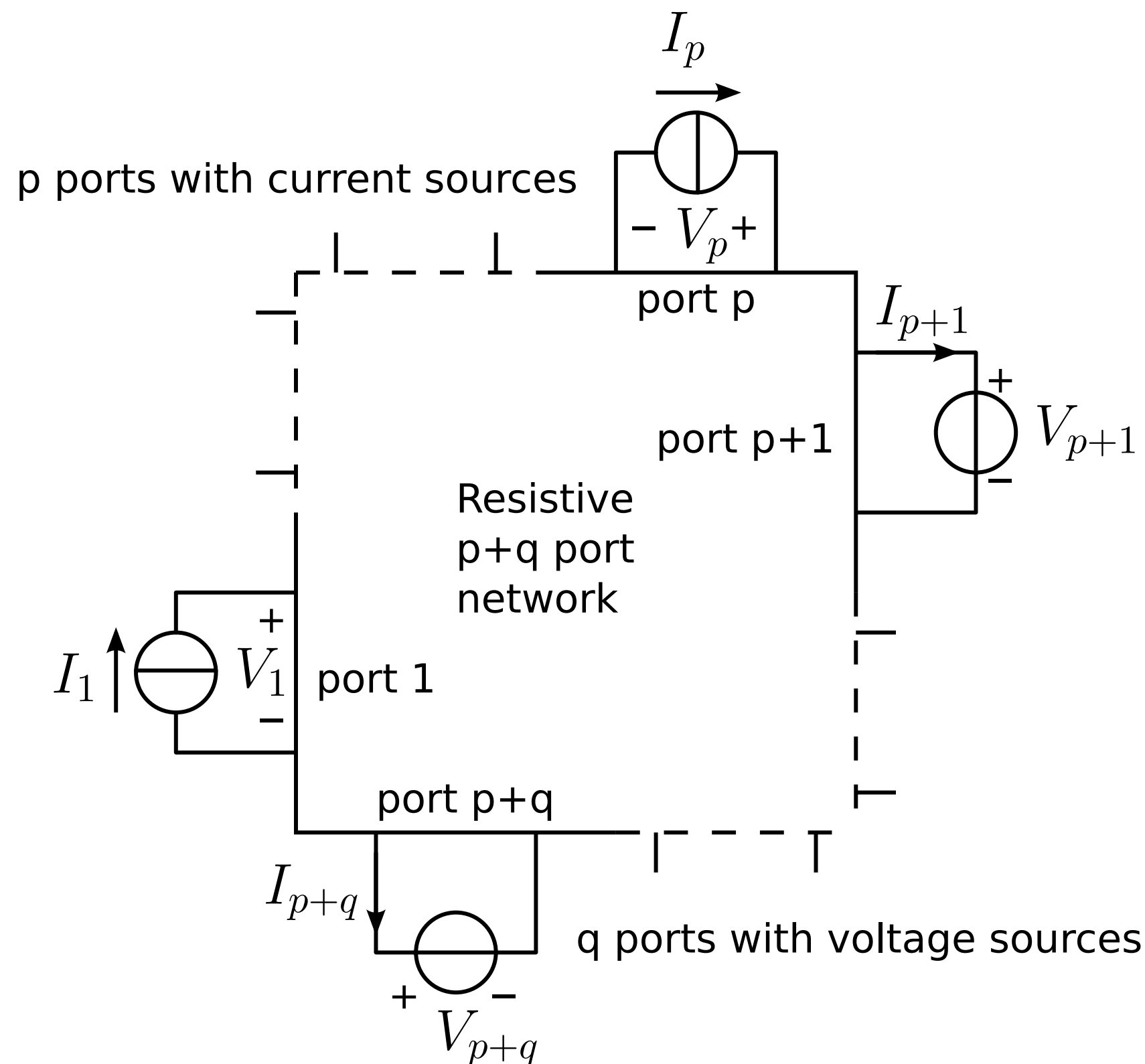
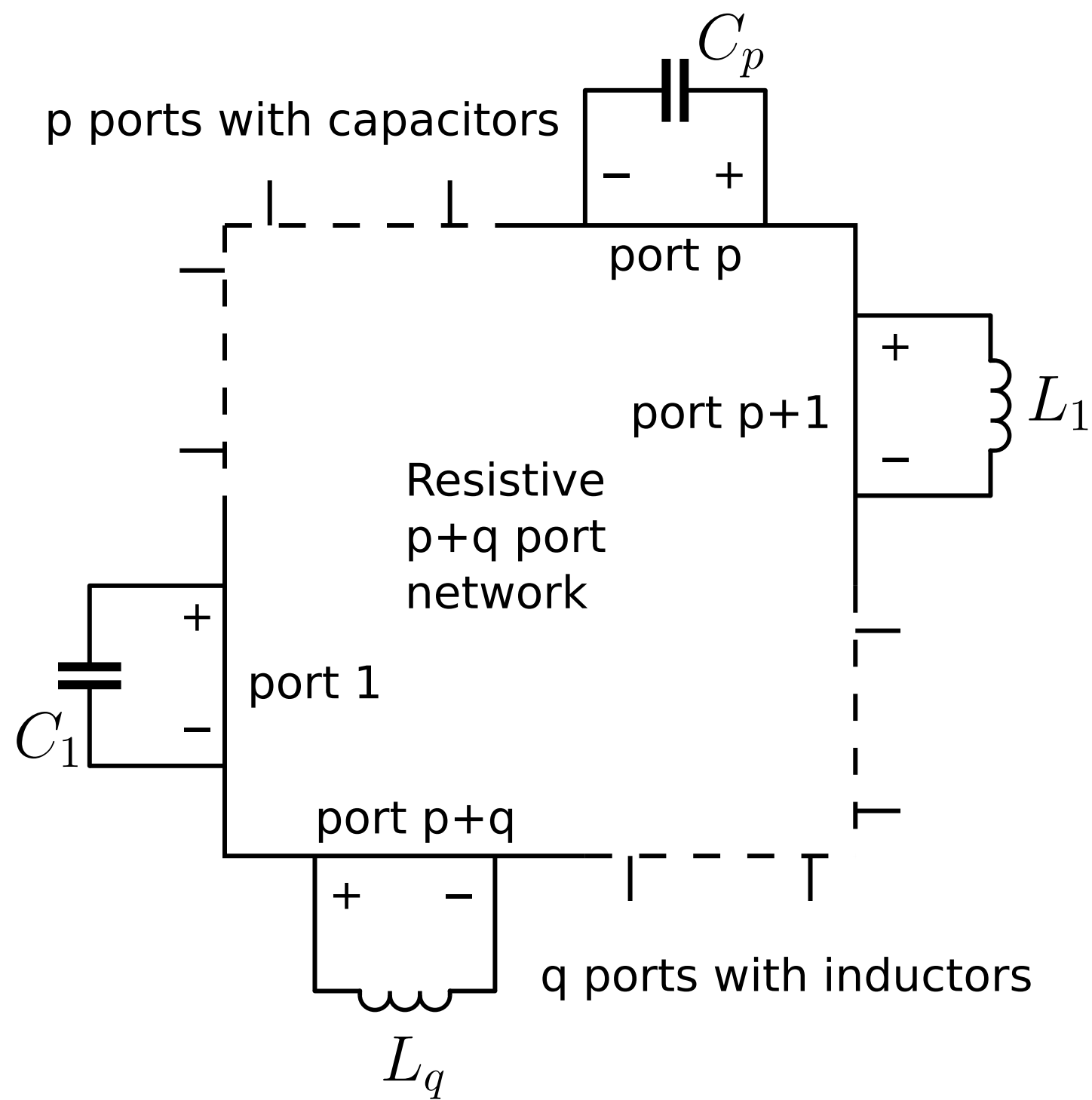


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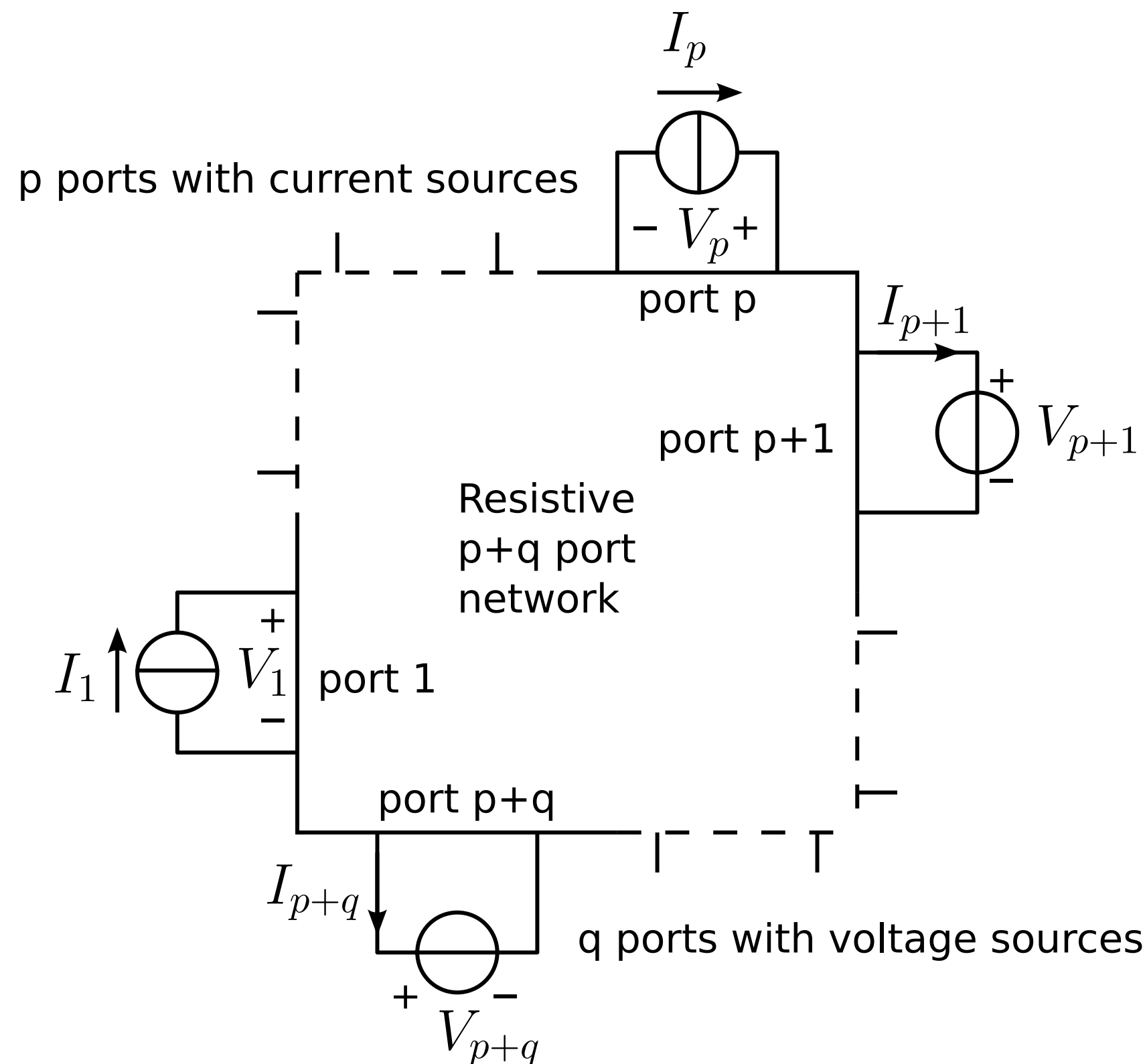


Resistance matrix relates dependent port variables to independent port variables

# Resistance matrix

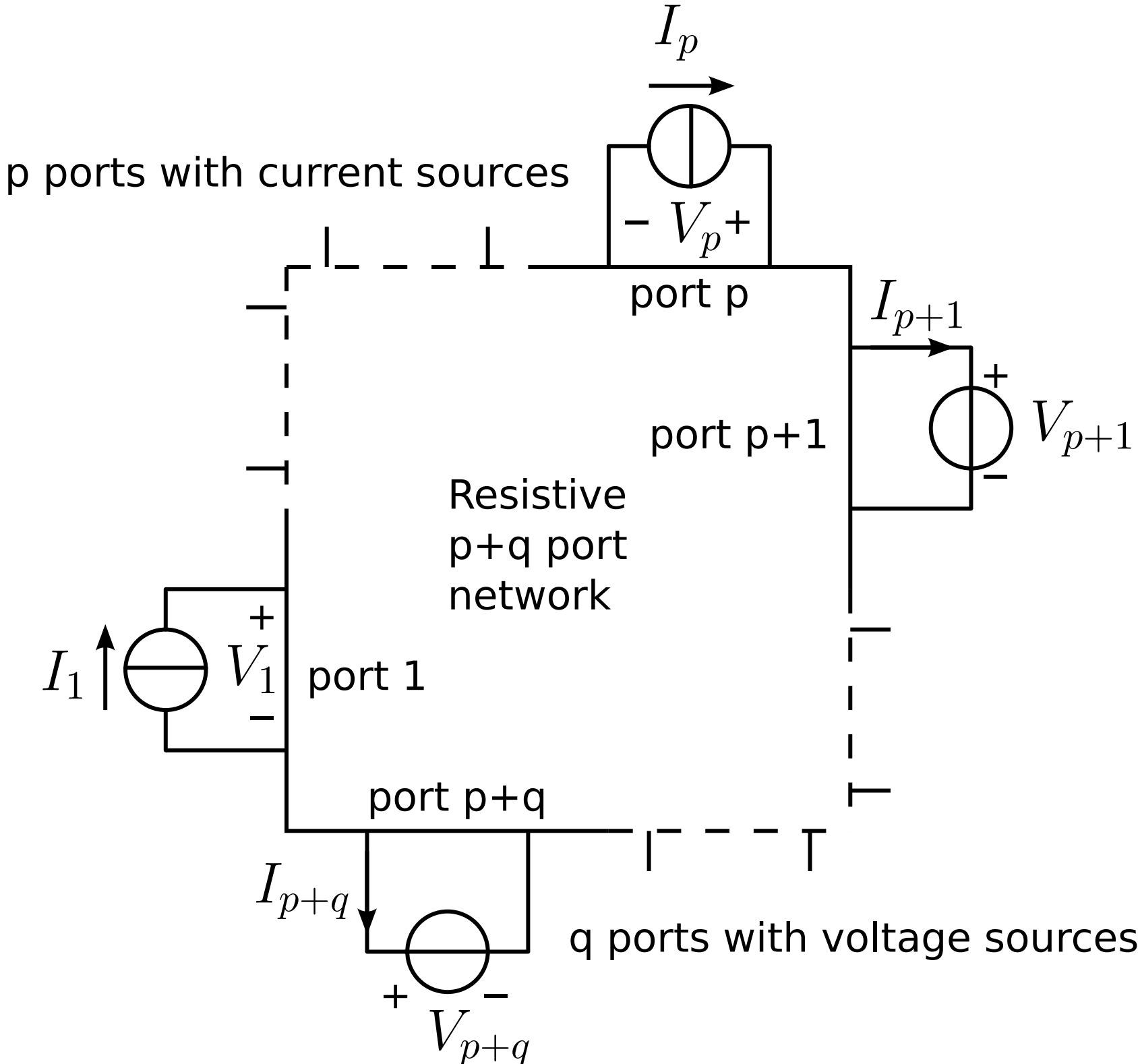


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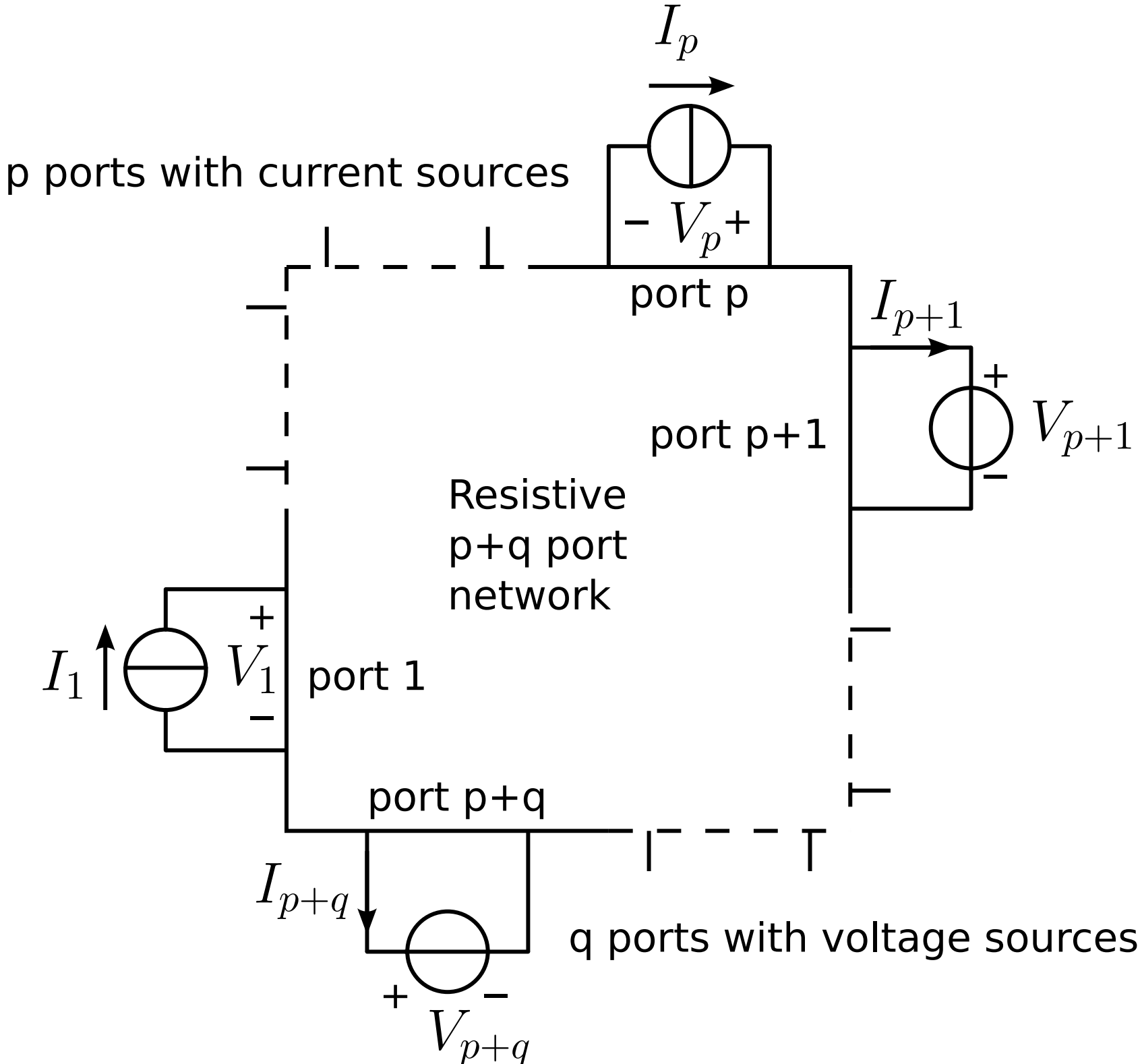


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Diagonal elements:  $\mathcal{R}_{i,i}$



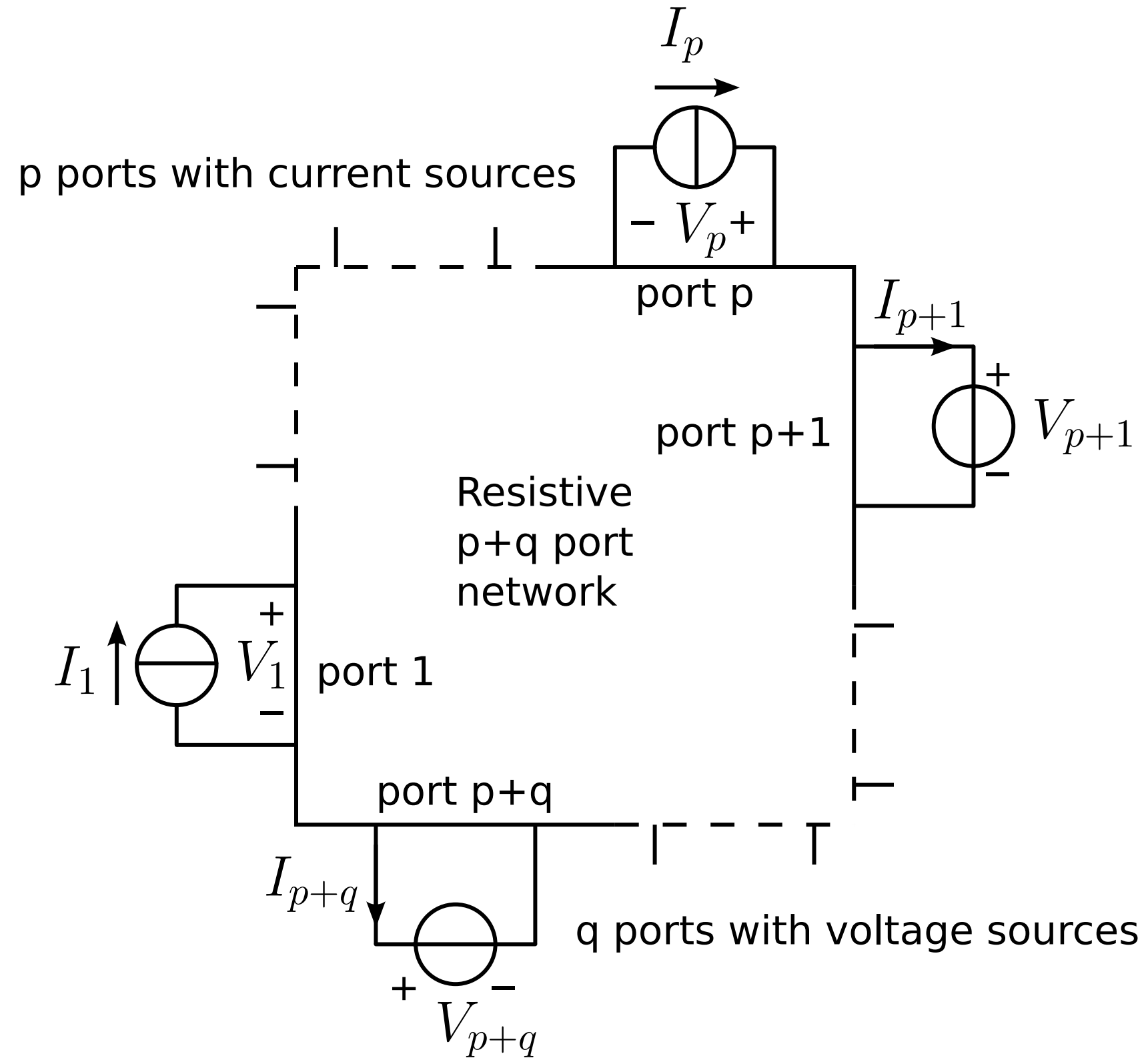
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Diagonal elements:  $\mathcal{R}_{i,i}$   
Resistance, conductance at port i

# Resistance matrix



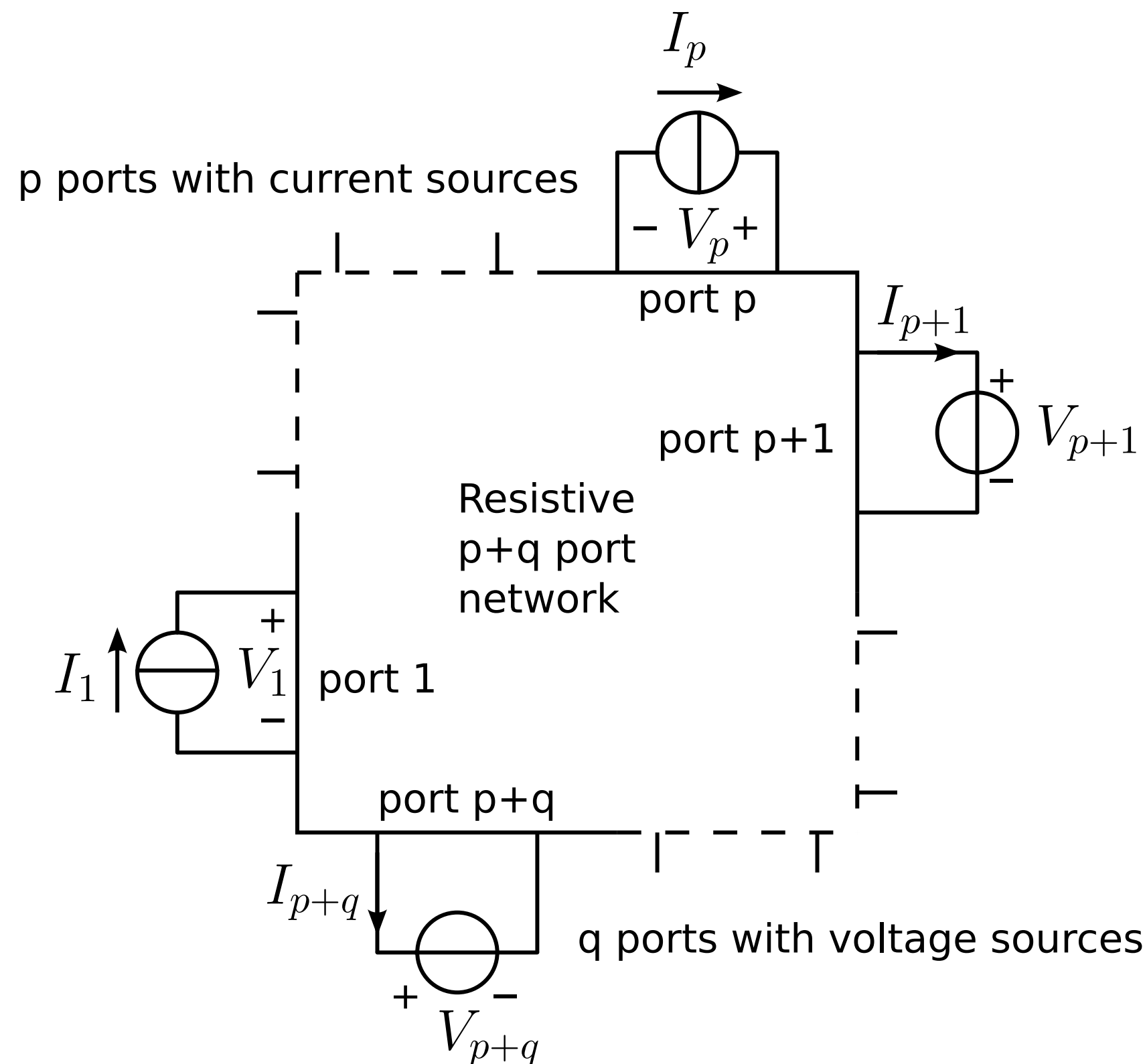
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Off-diagonal elements:  $\mathcal{R}_{i,j}$

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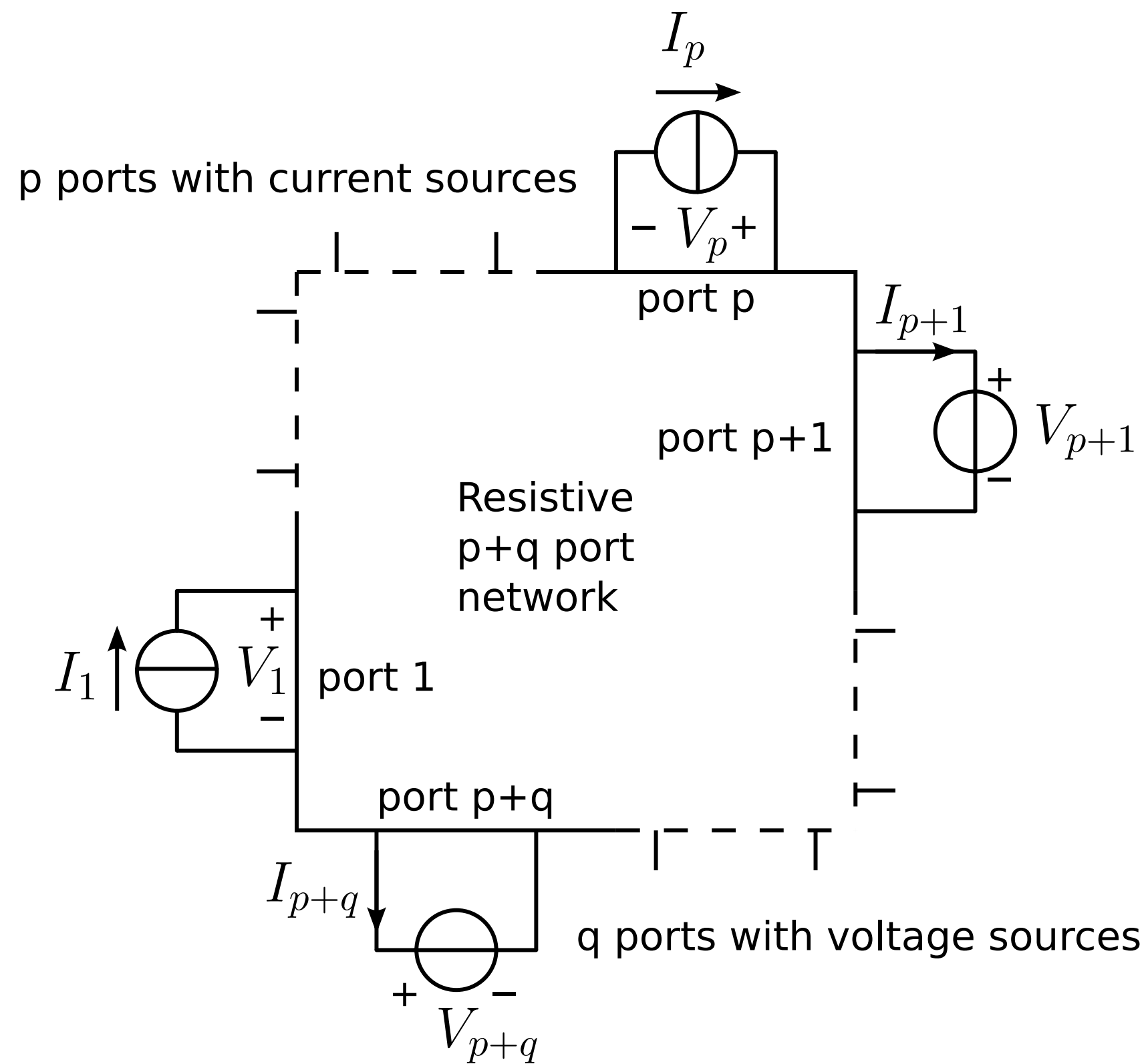


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Off-diagonal elements:  $\mathcal{R}_{i,j}$   
transresistance

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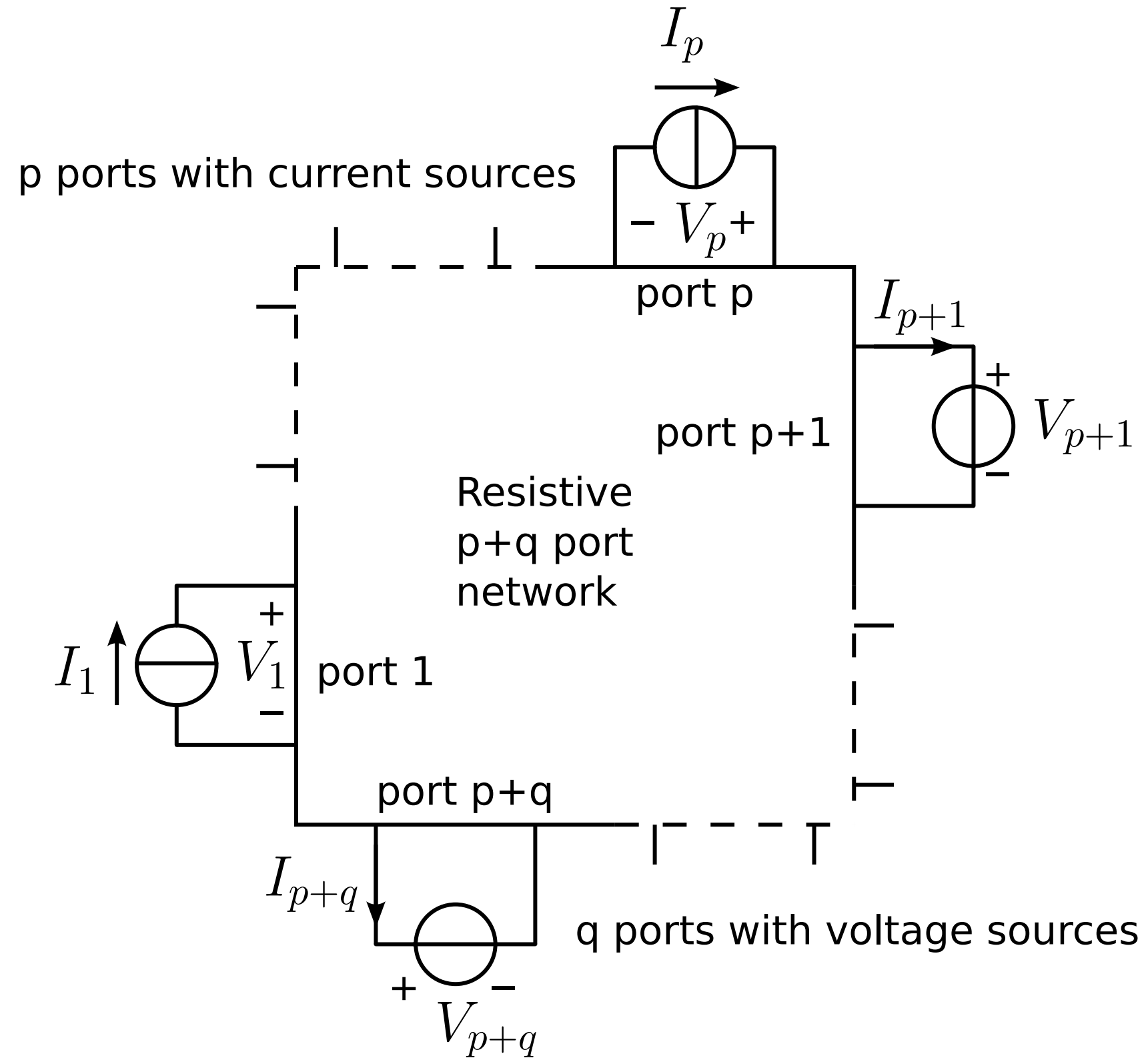
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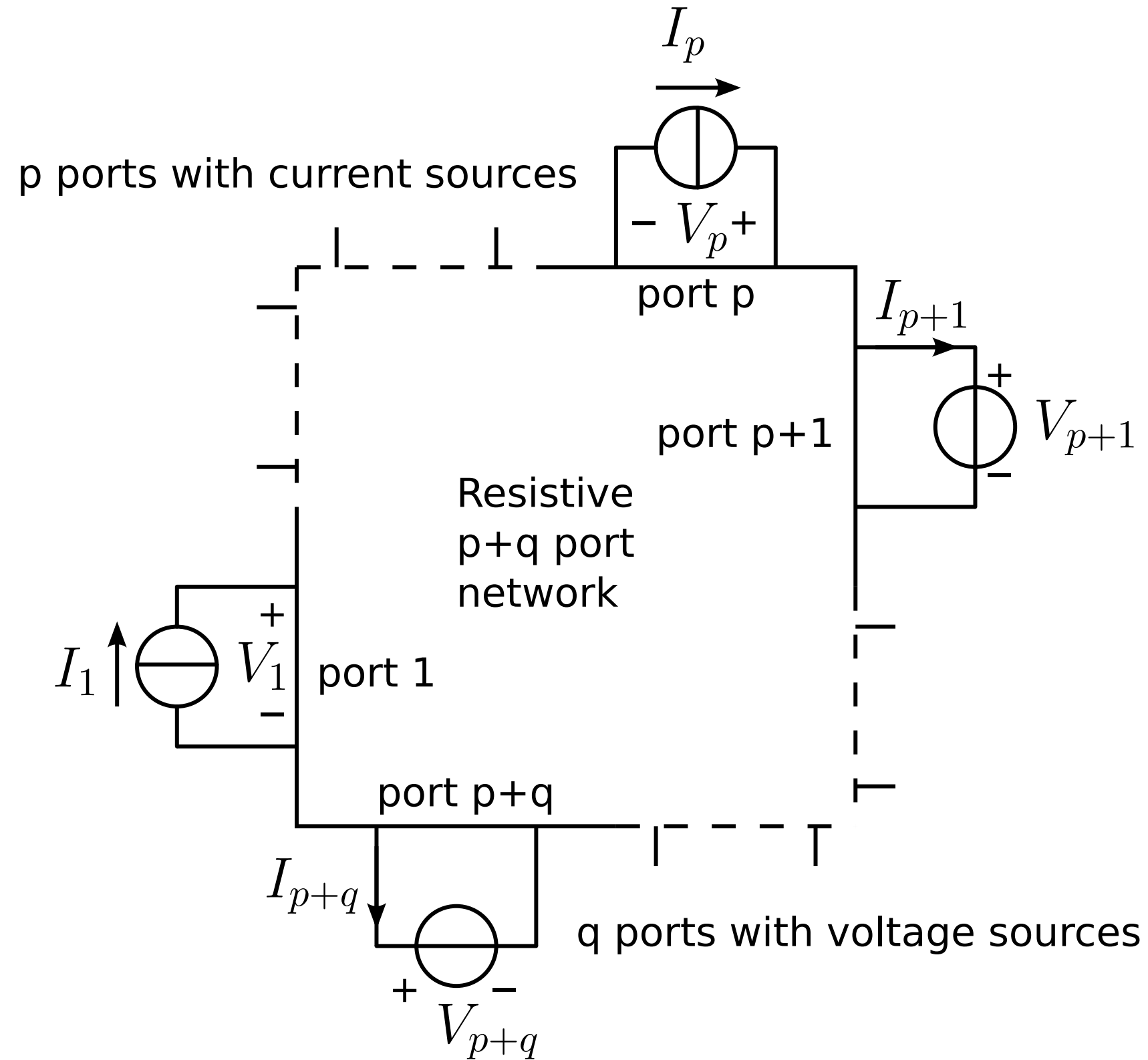
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voltage gain factor

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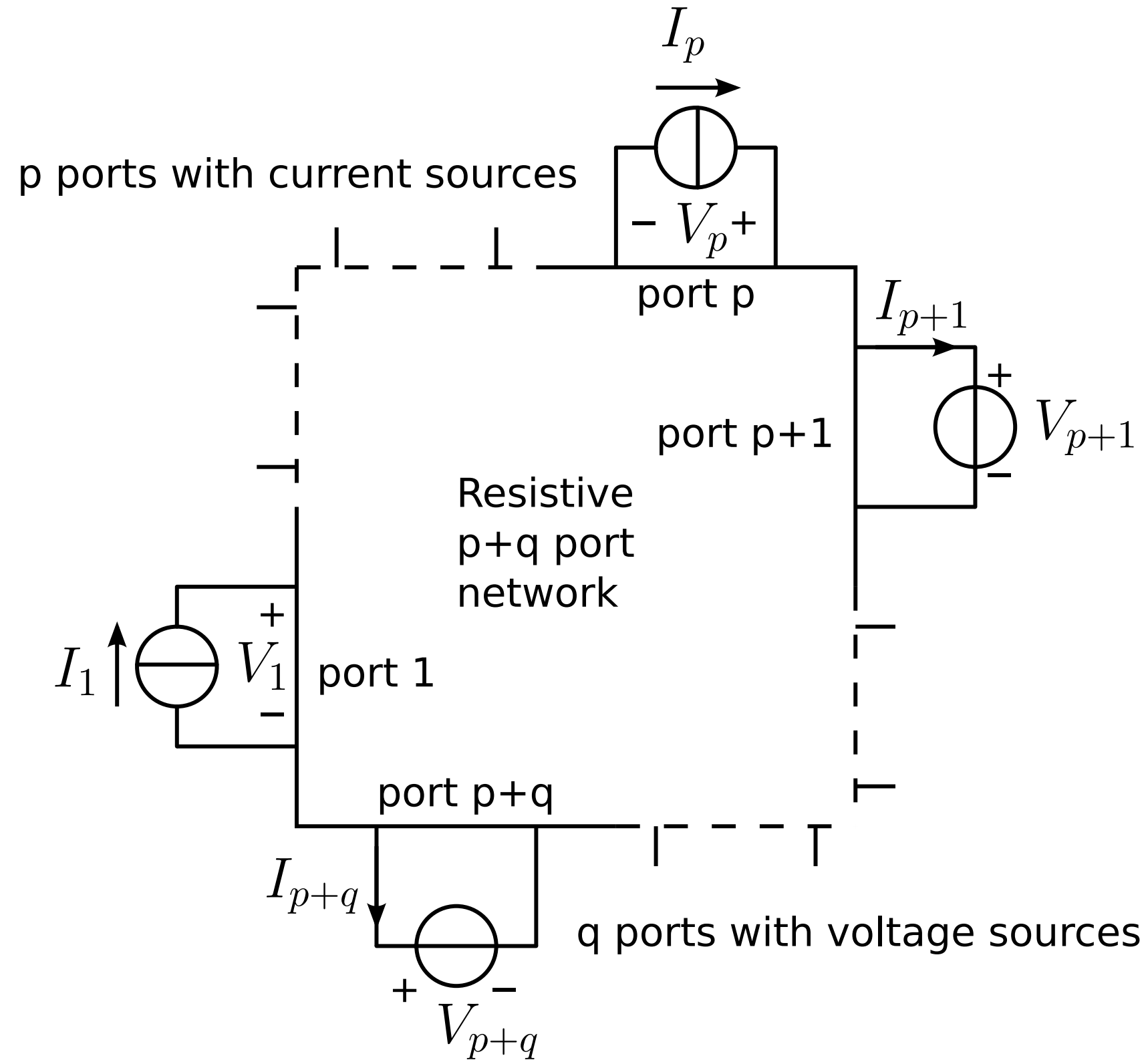
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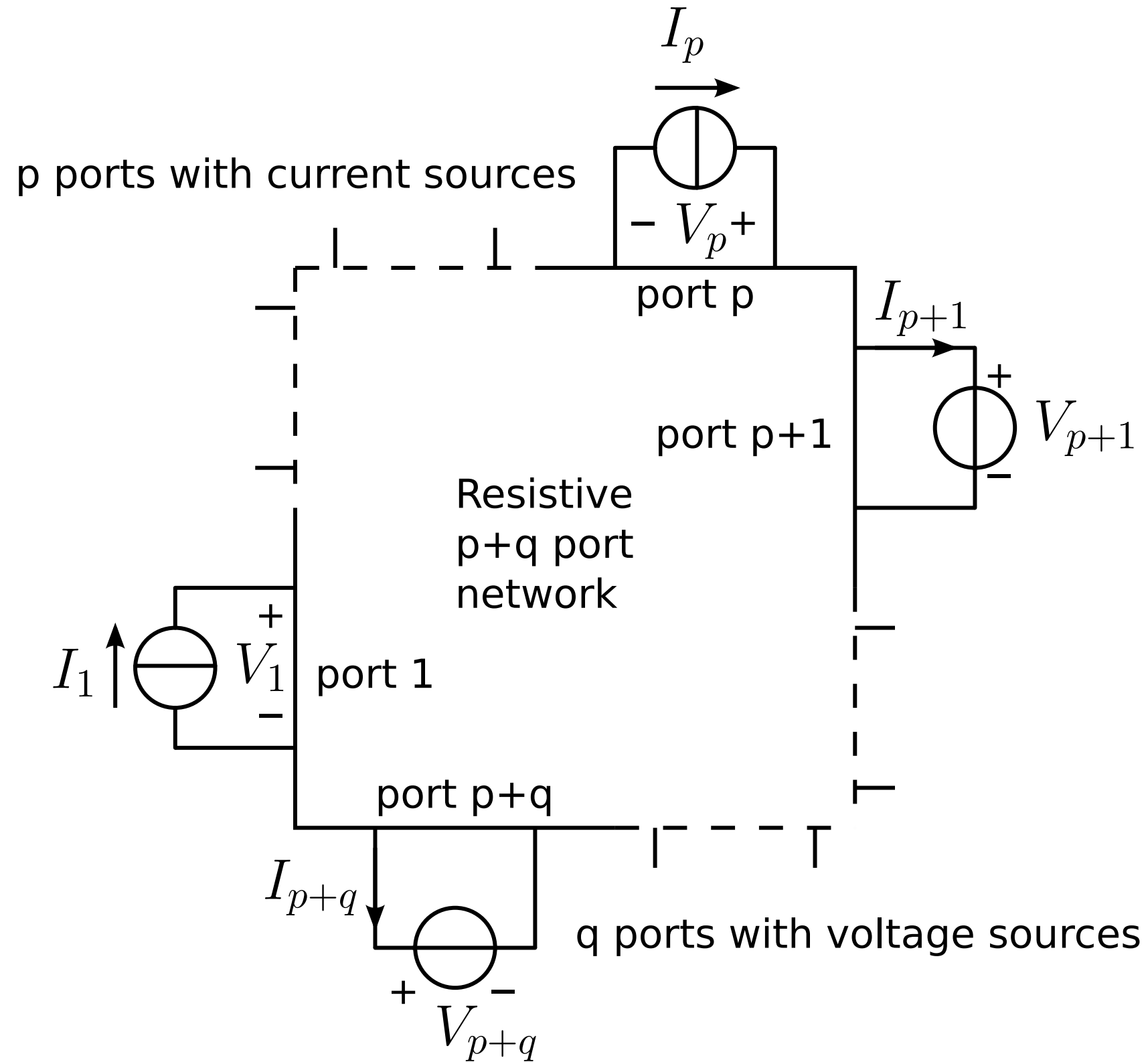
transconductance

voltage gain factor

current gain factor

from port i to port j

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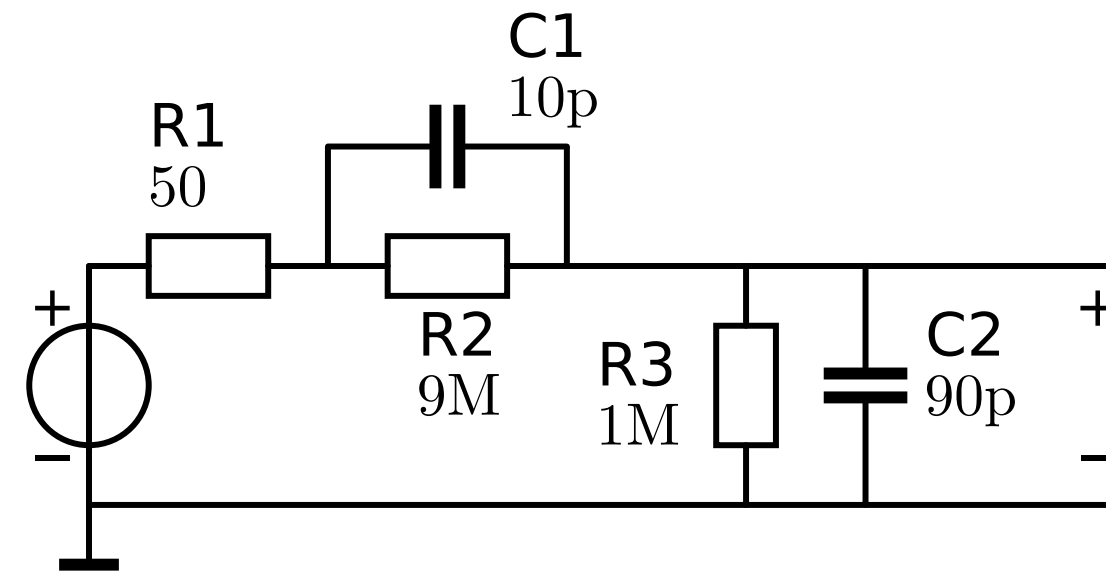
current gain factor

from port i to port j

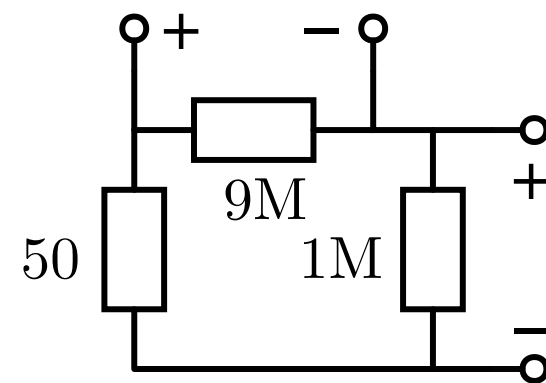


# RC matrix example

## Oscilloscope probe circuit



## R-port



## R-matrix

## C-matrix

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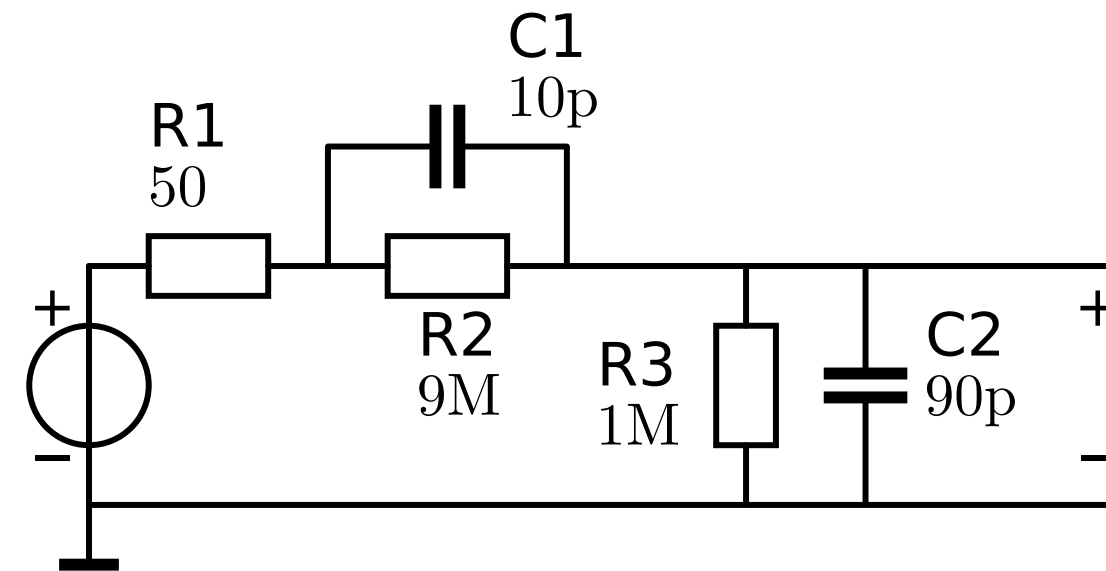
Poles from eigenvalues of RC:

$$p_1 = -1768 \text{ Hz}$$

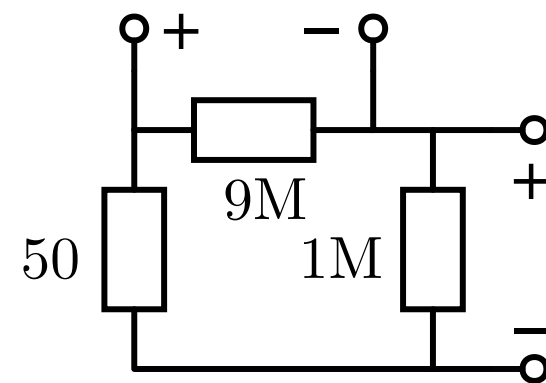
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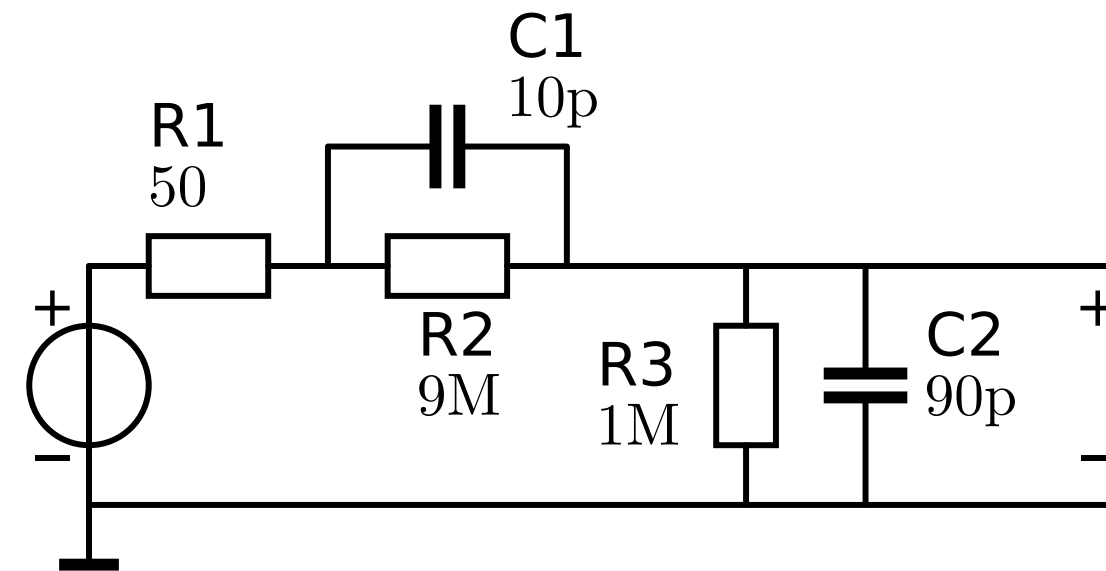
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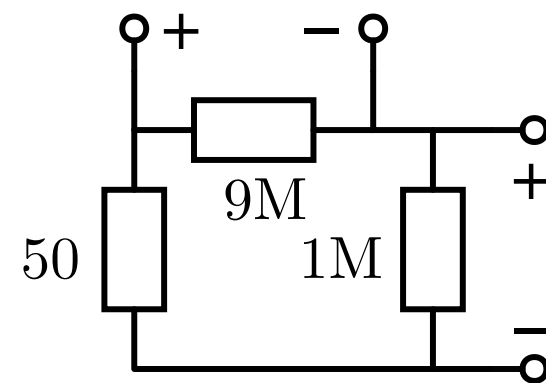
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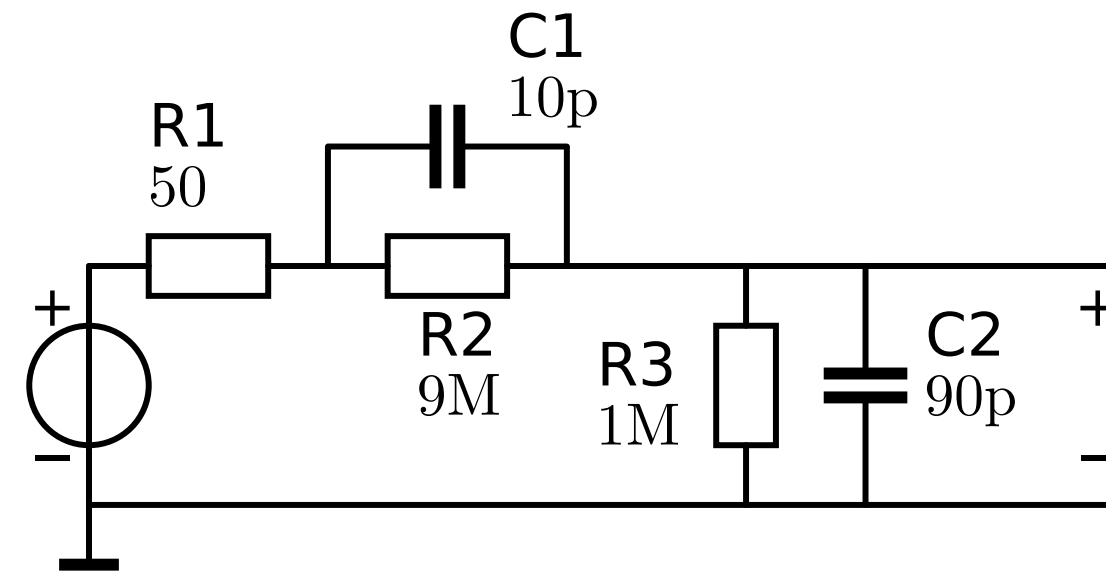
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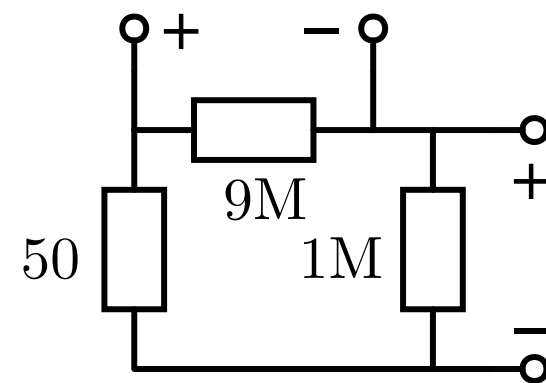
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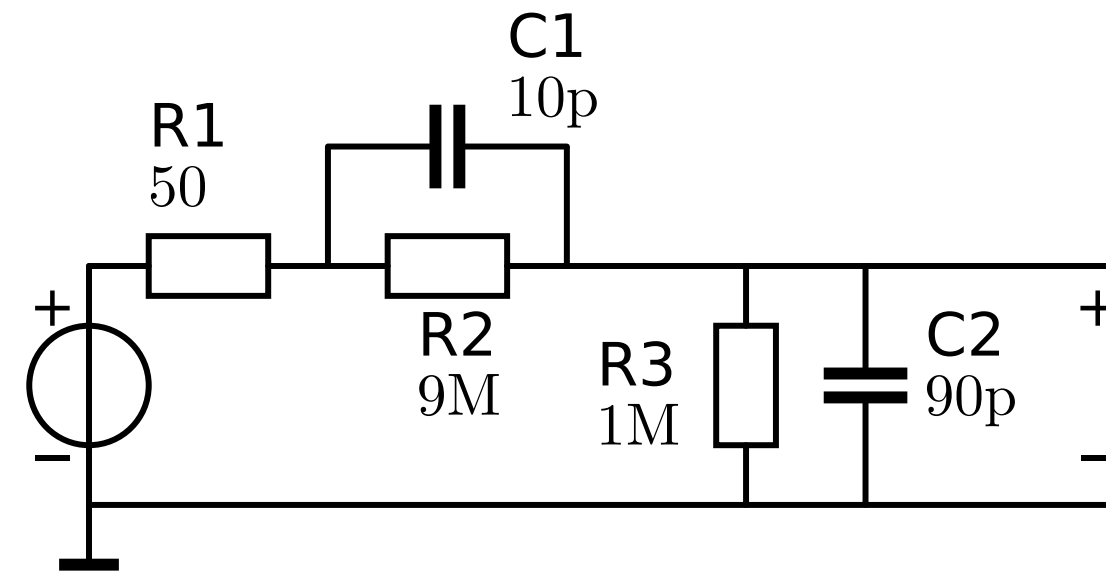
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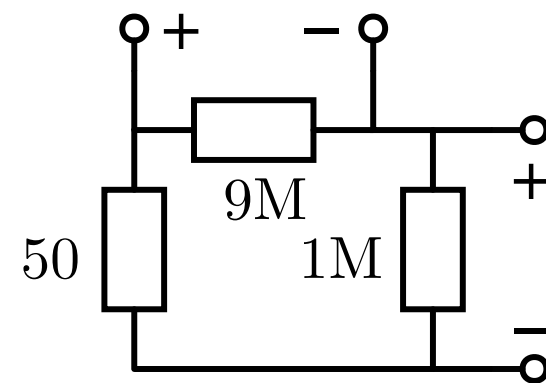
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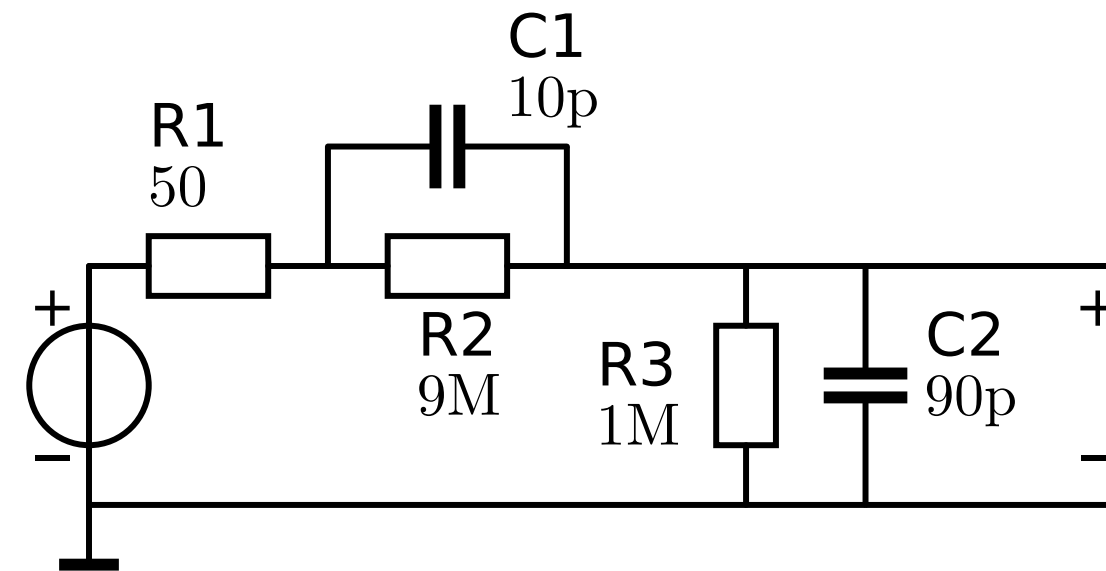
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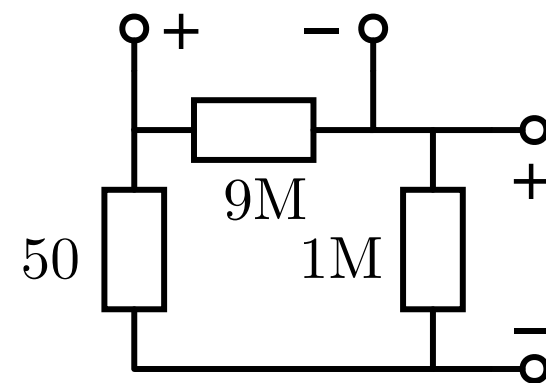
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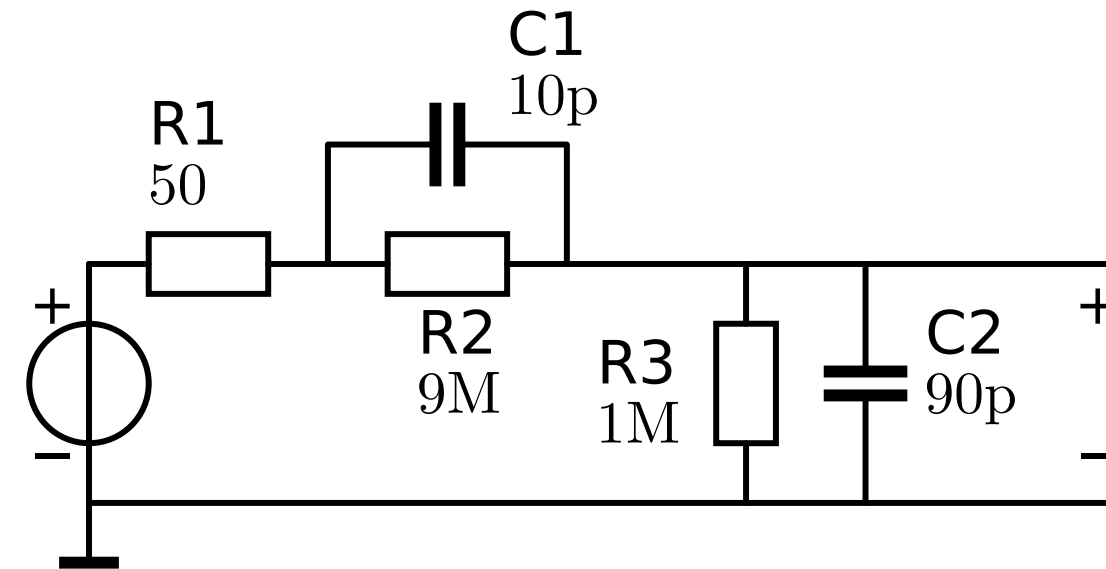
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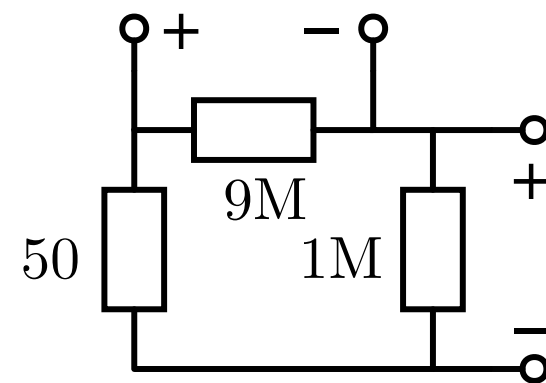
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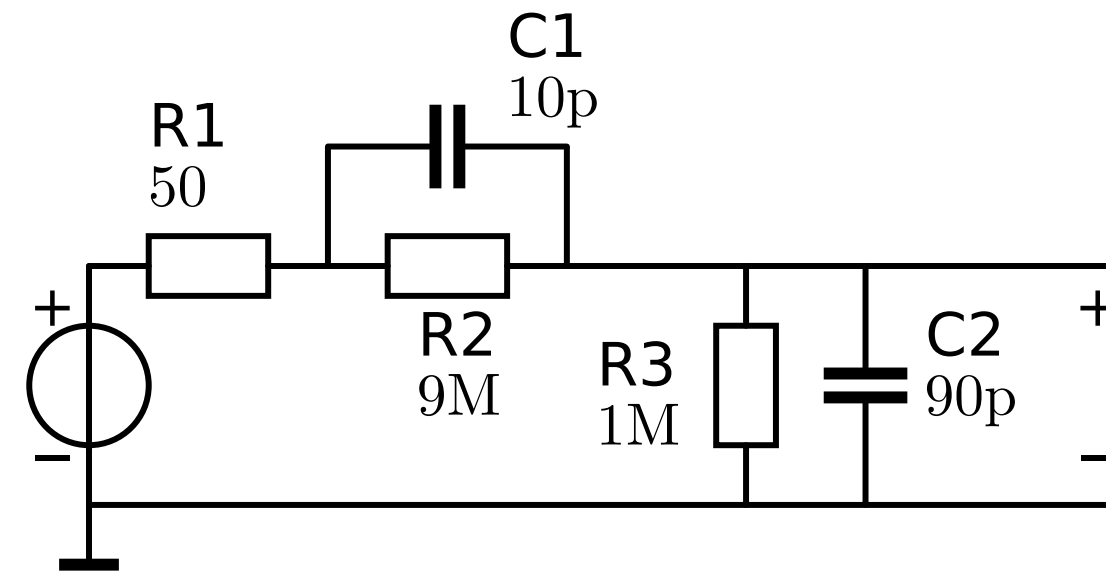
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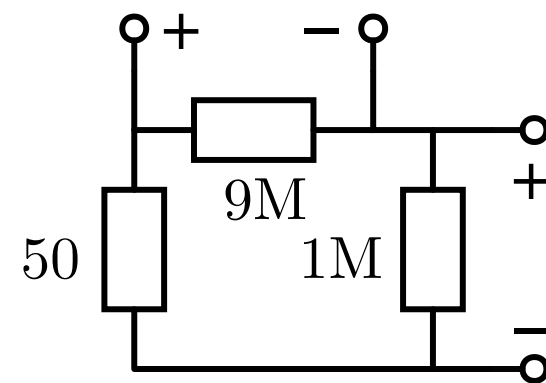


# RC matrix example

## Oscilloscope probe circuit



### R-port



### R-matrix

### C-matrix

$$\begin{pmatrix} \frac{9 \cdot 10^6 (50 + 10^6)}{50 + 10 \cdot 10^6} & -\frac{9 \cdot 10^6 \times 10^6}{50 + 10 \cdot 10^6} \\ -\frac{9 \cdot 10^6 \times 10^6}{50 + 10 \cdot 10^6} & \frac{10^6 (50 + 9 \cdot 10^6)}{50 + 10 \cdot 10^6} \end{pmatrix} \begin{pmatrix} 10^{-11} & 0 \\ 0 & 90 \cdot 10^{-12} \end{pmatrix}$$

Poles from eigenvalues of RC:

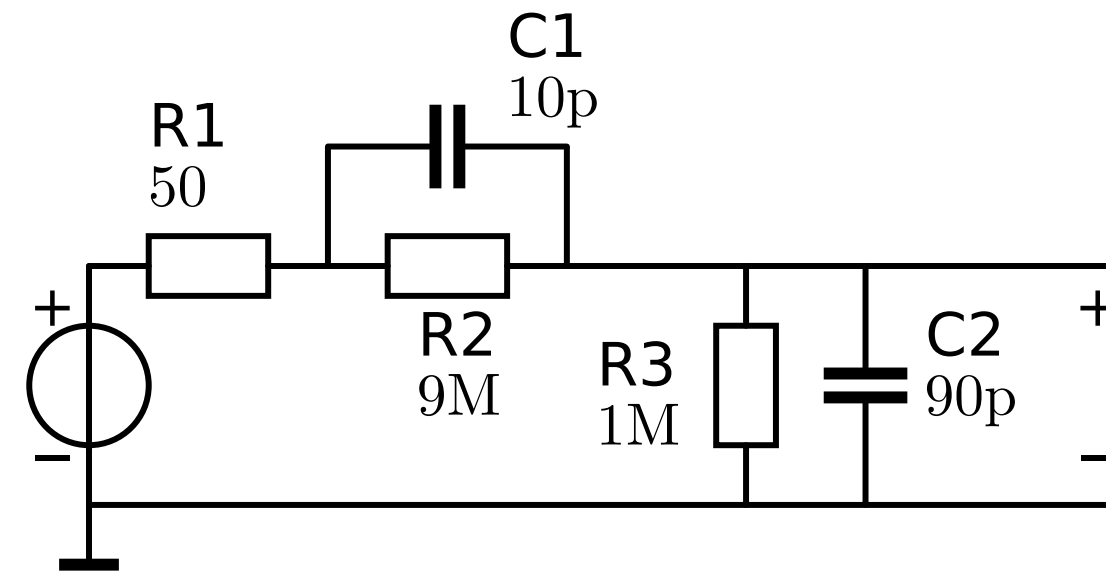
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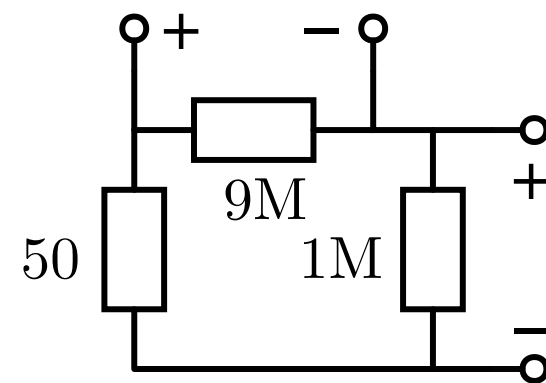


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# Estimation of the poles from the diagonal elements of the RC matrix

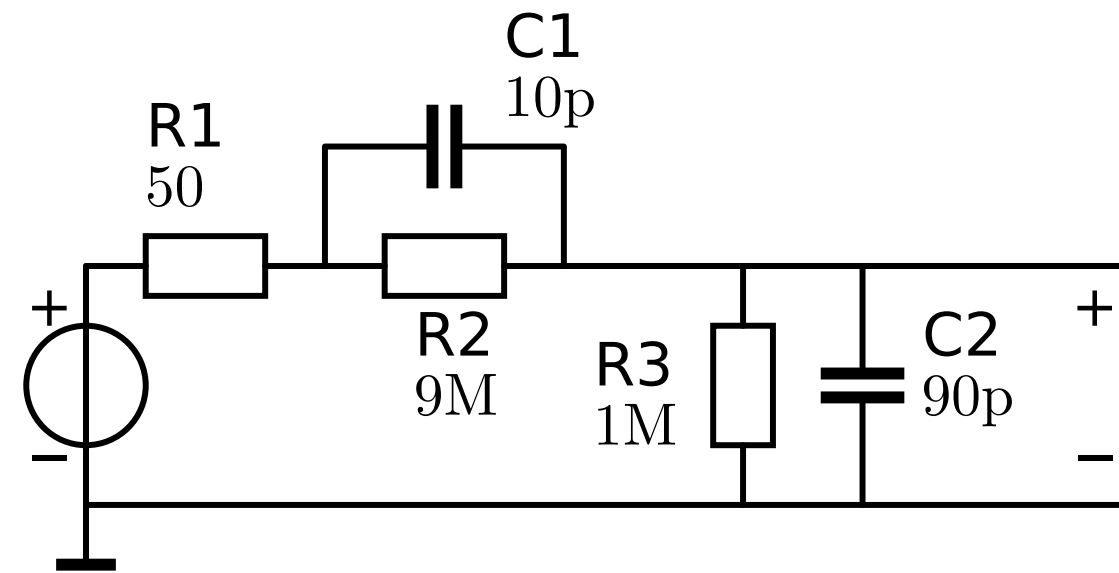
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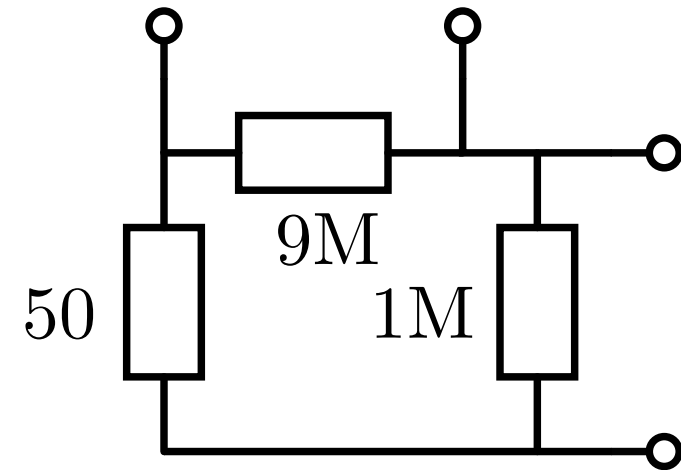
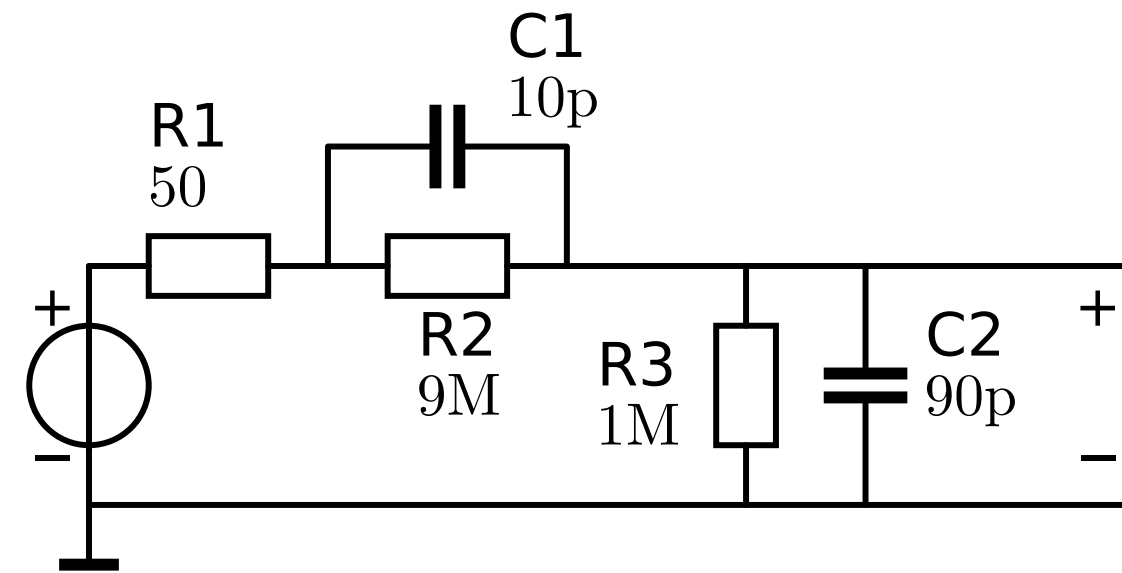
Oscilloscope probe circuit



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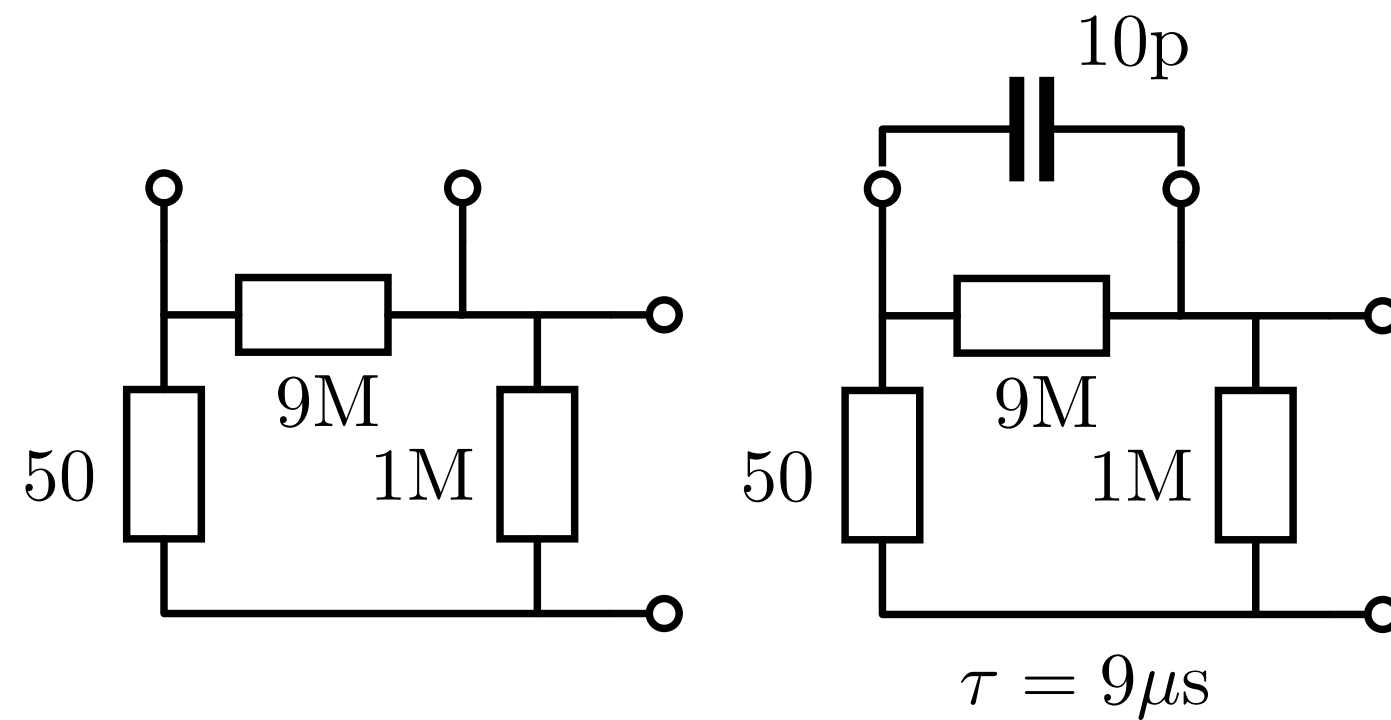
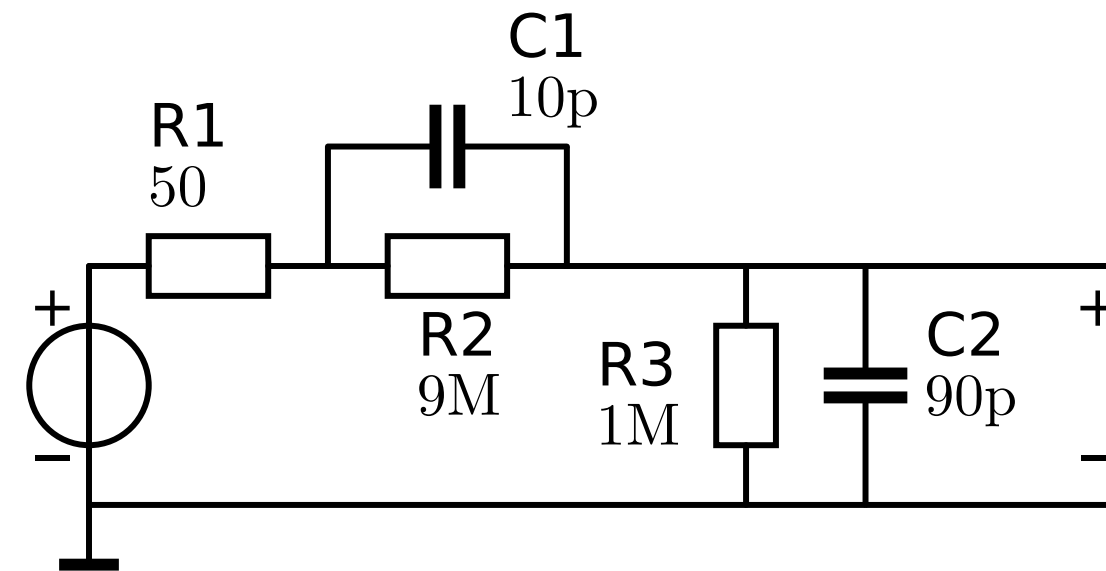
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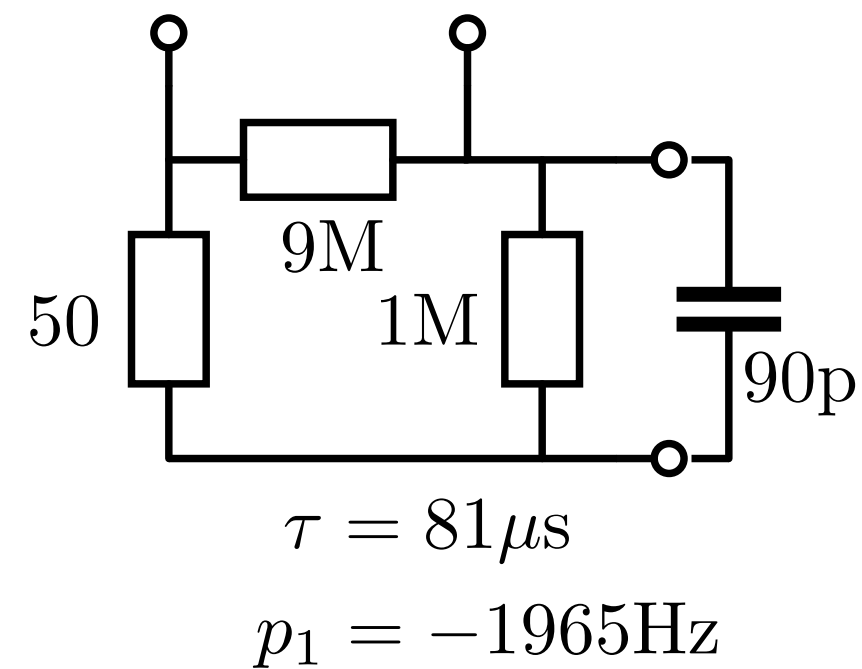
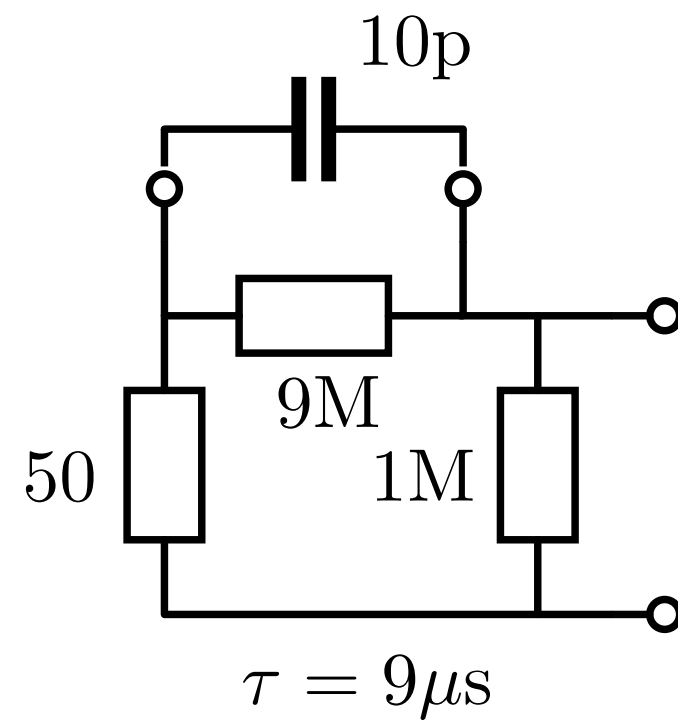
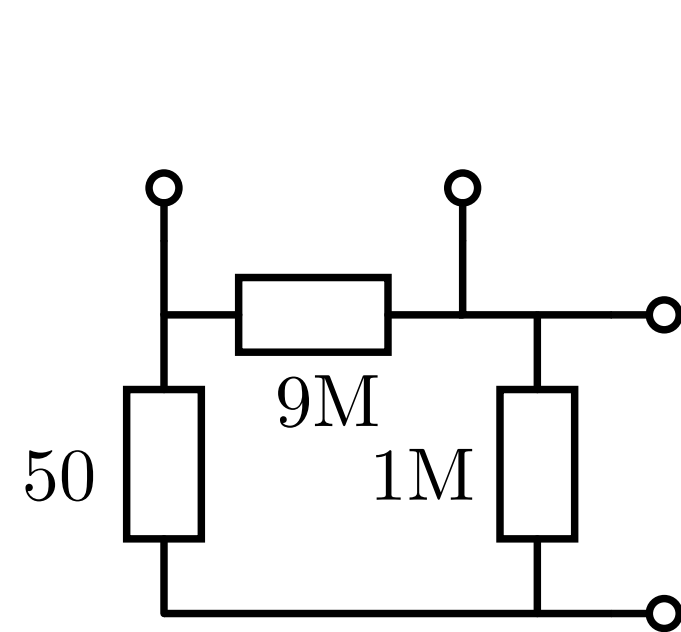
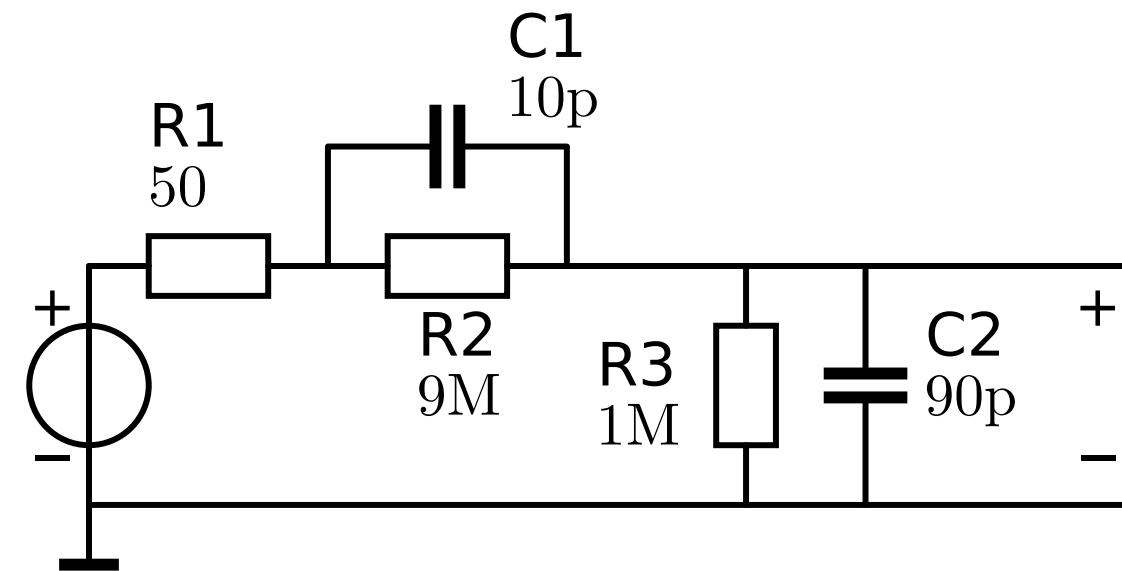
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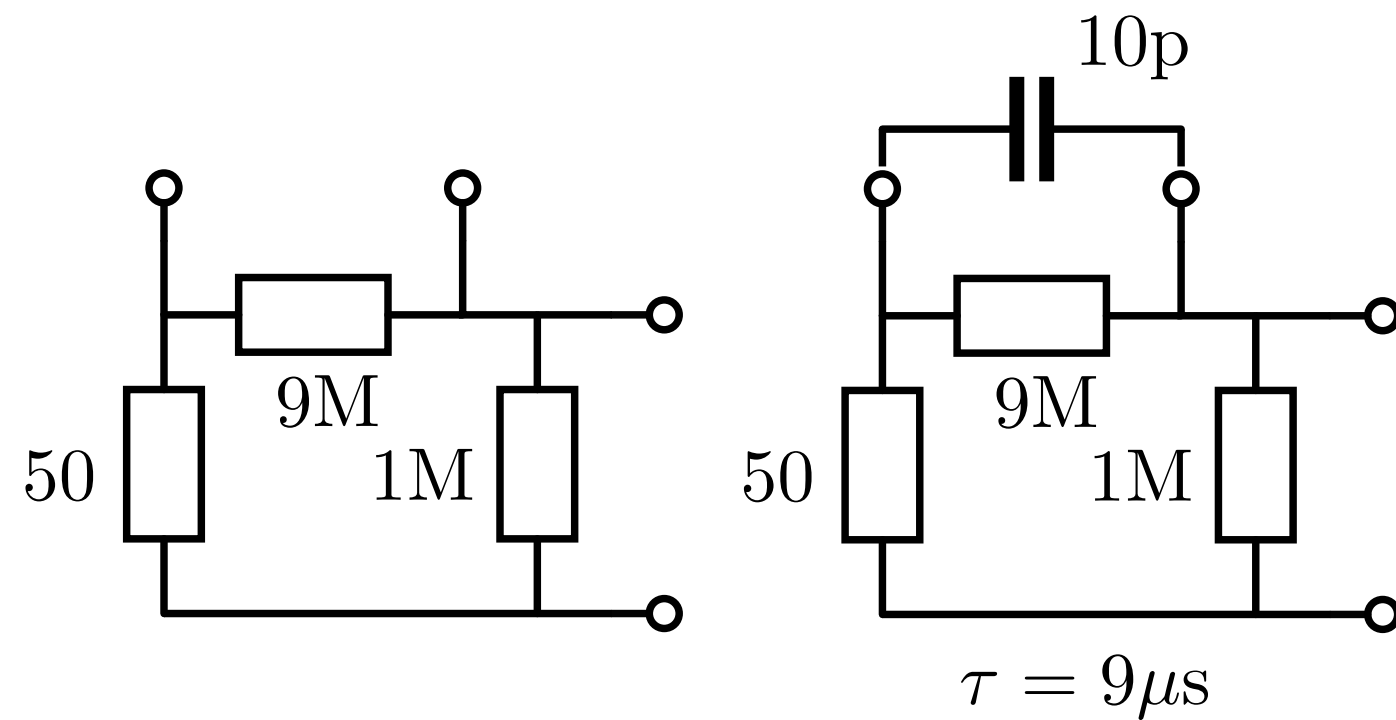
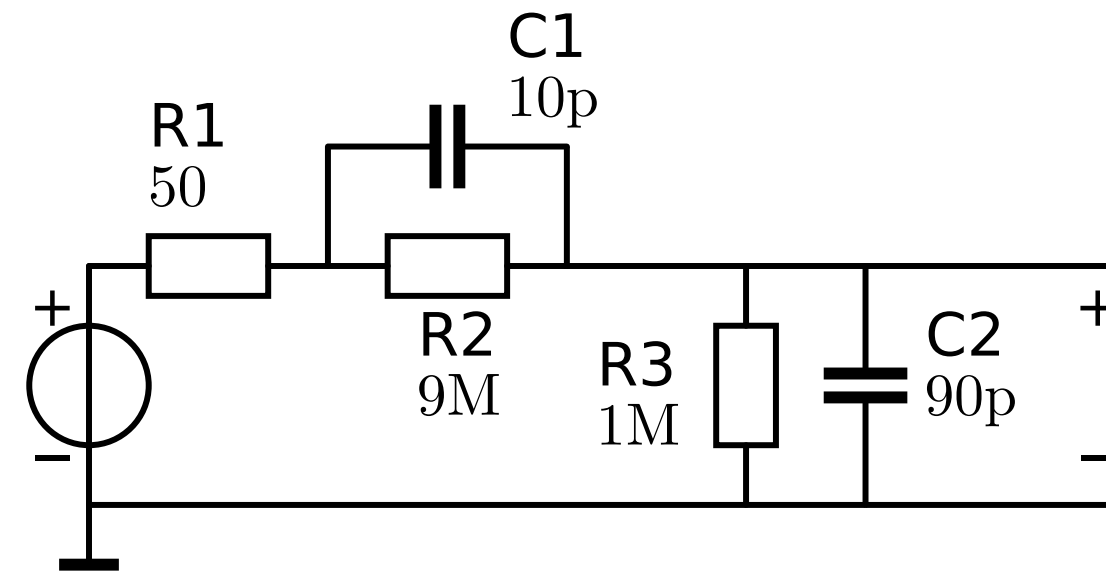
Oscilloscope probe circuit



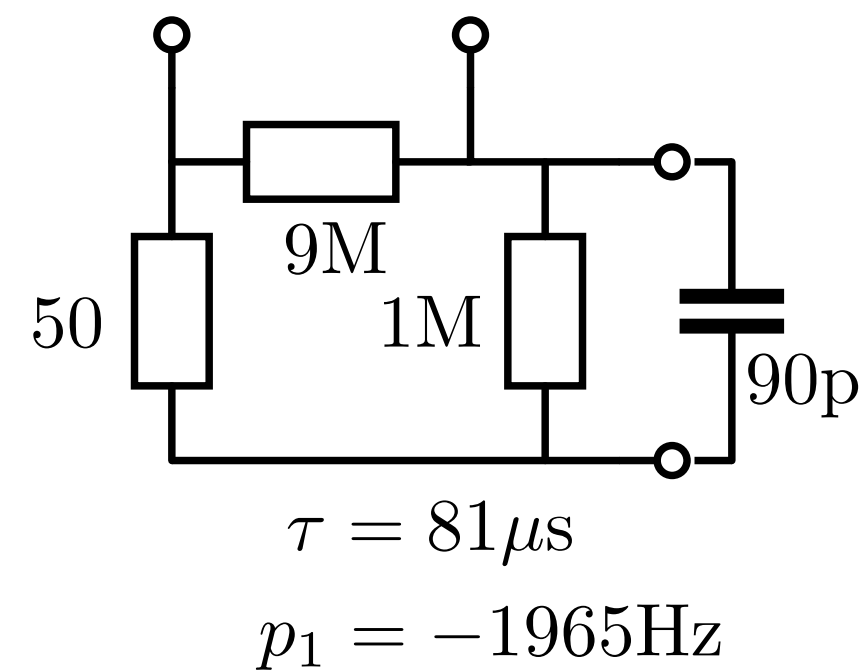
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Oscilloscope probe circuit



dominant pole

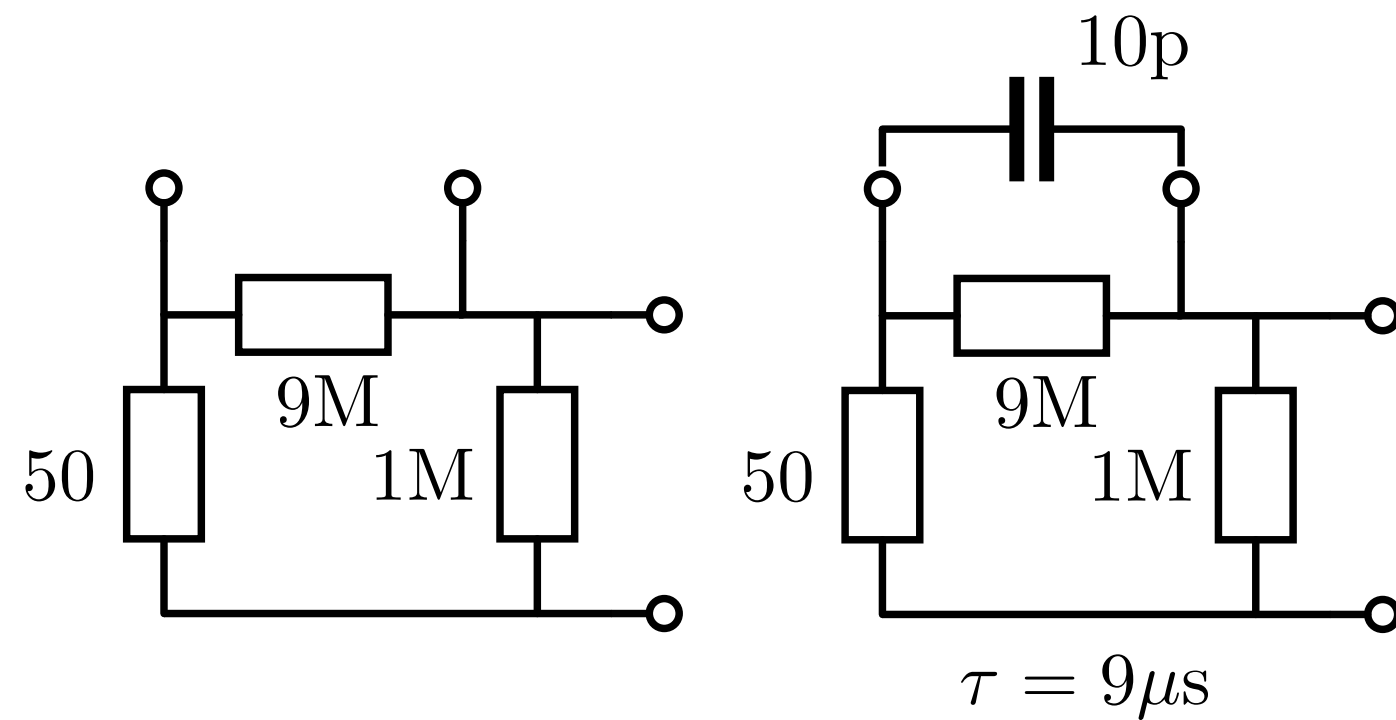
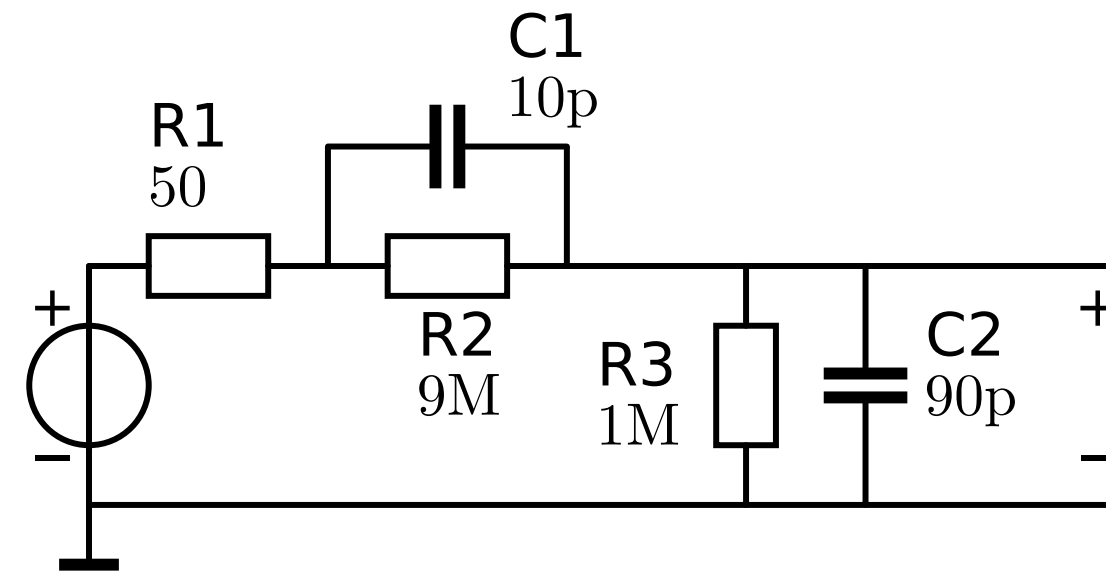




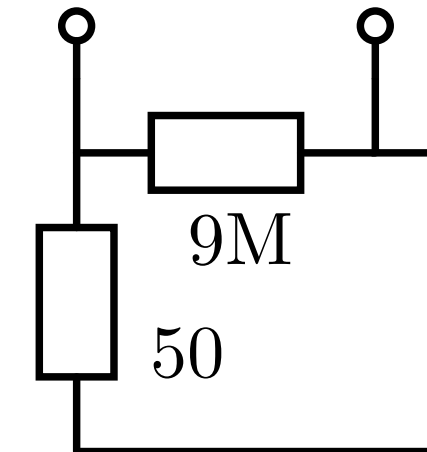
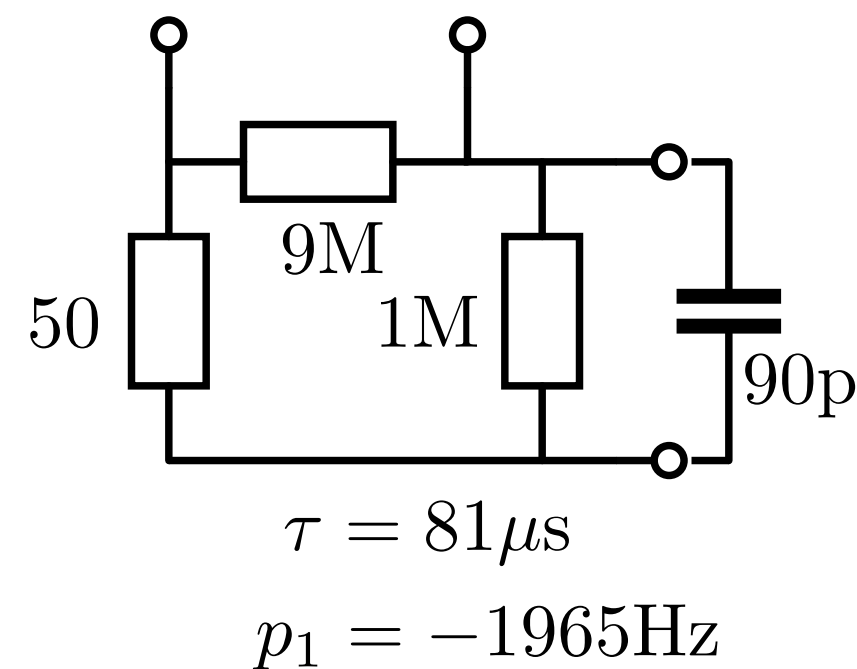
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Oscilloscope probe circuit



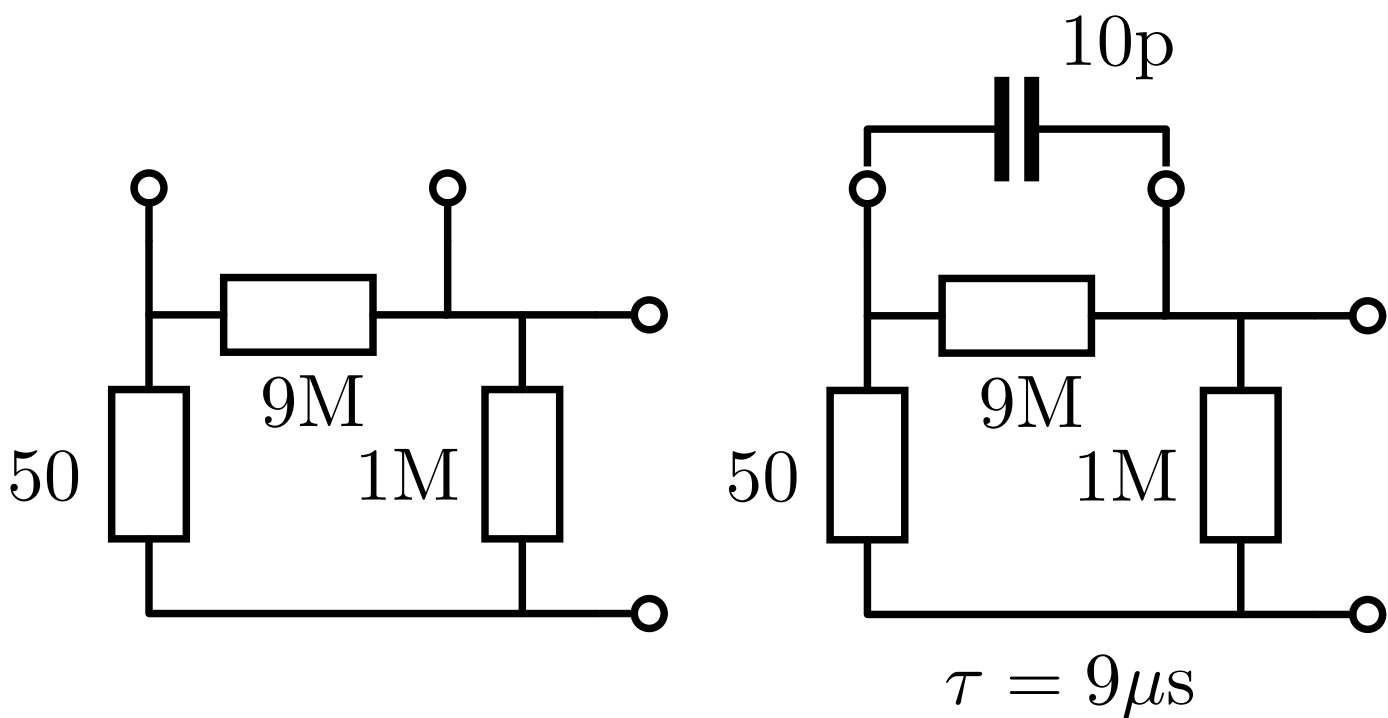
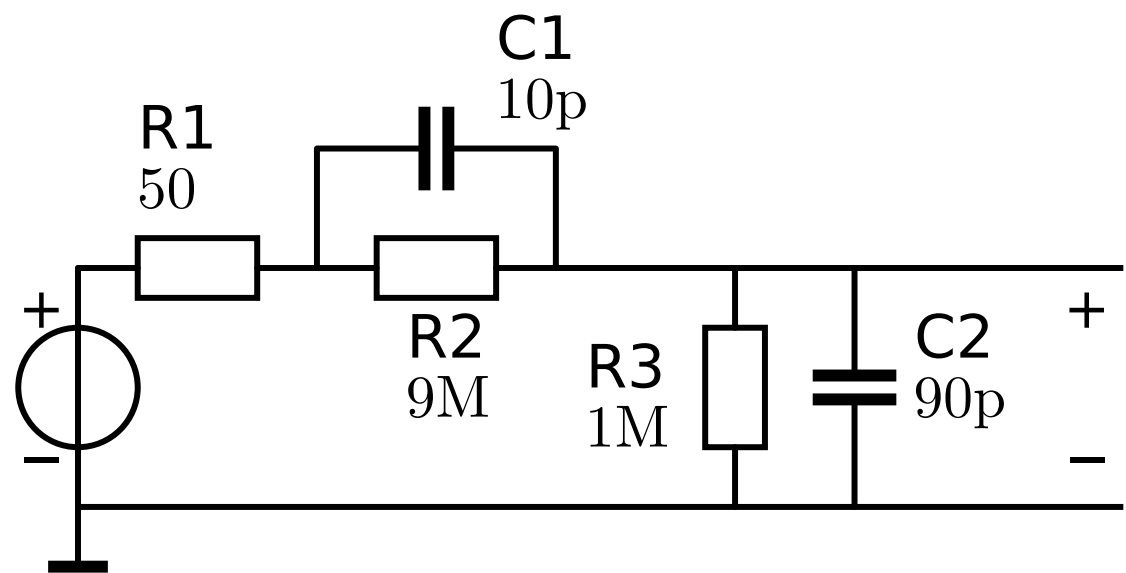
dominant pole



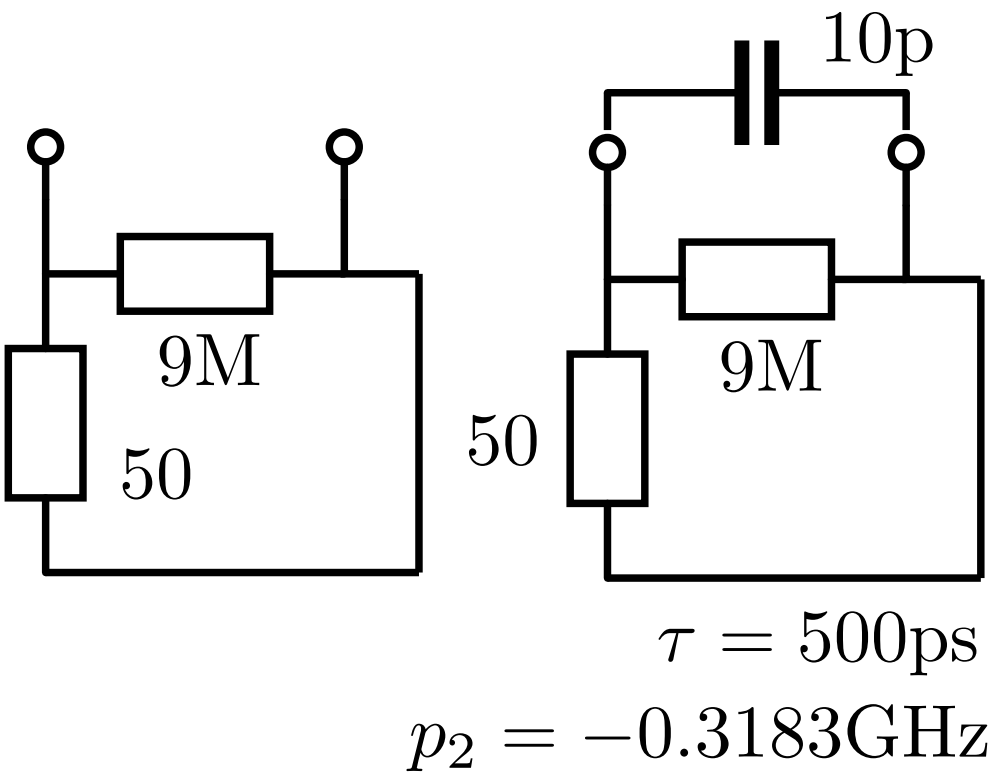
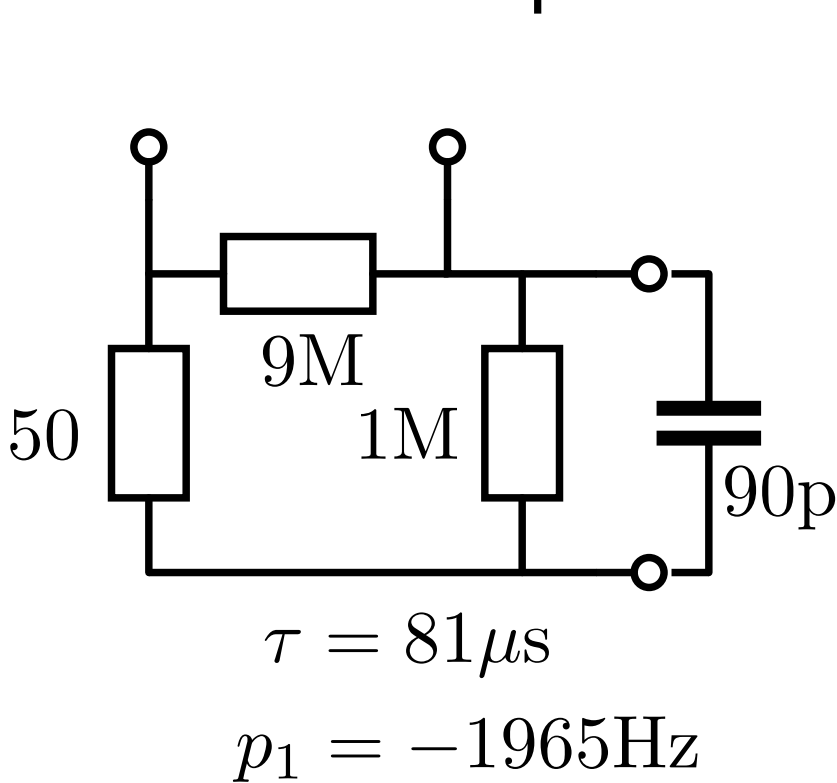
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Oscilloscope probe circuit



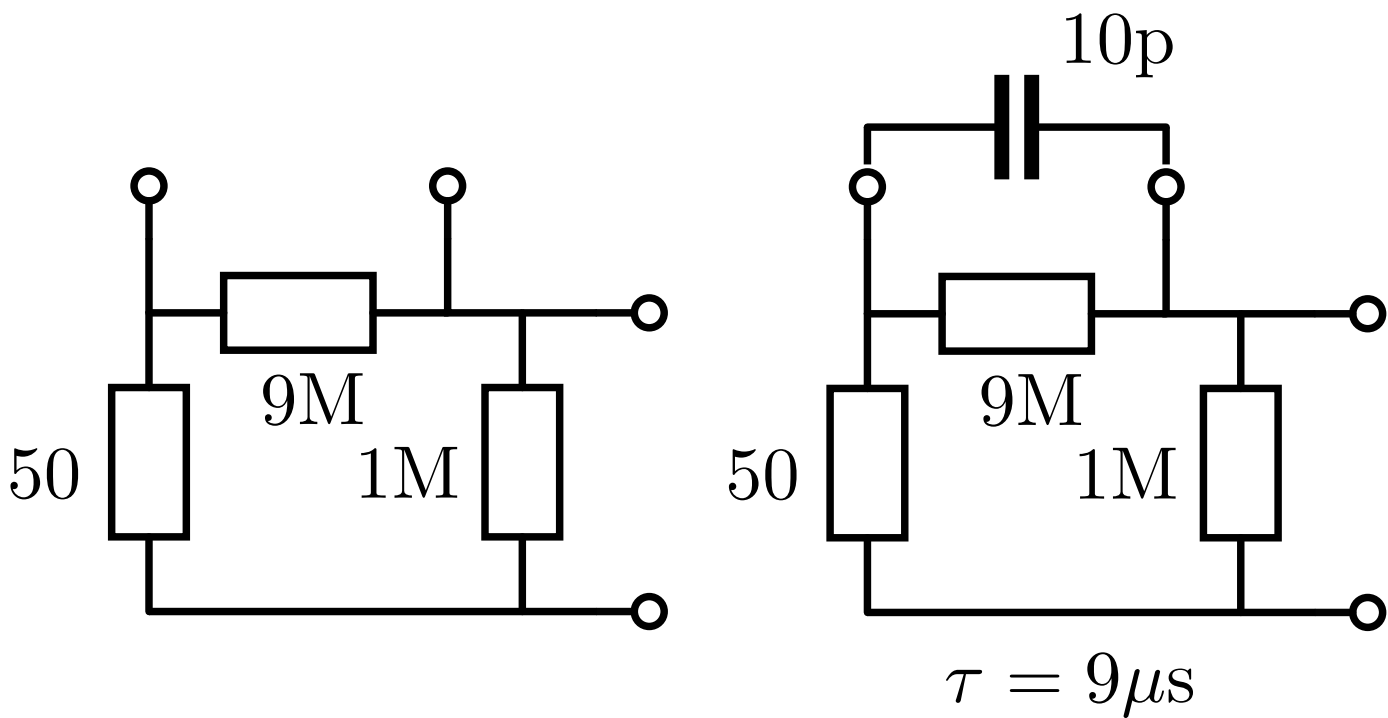
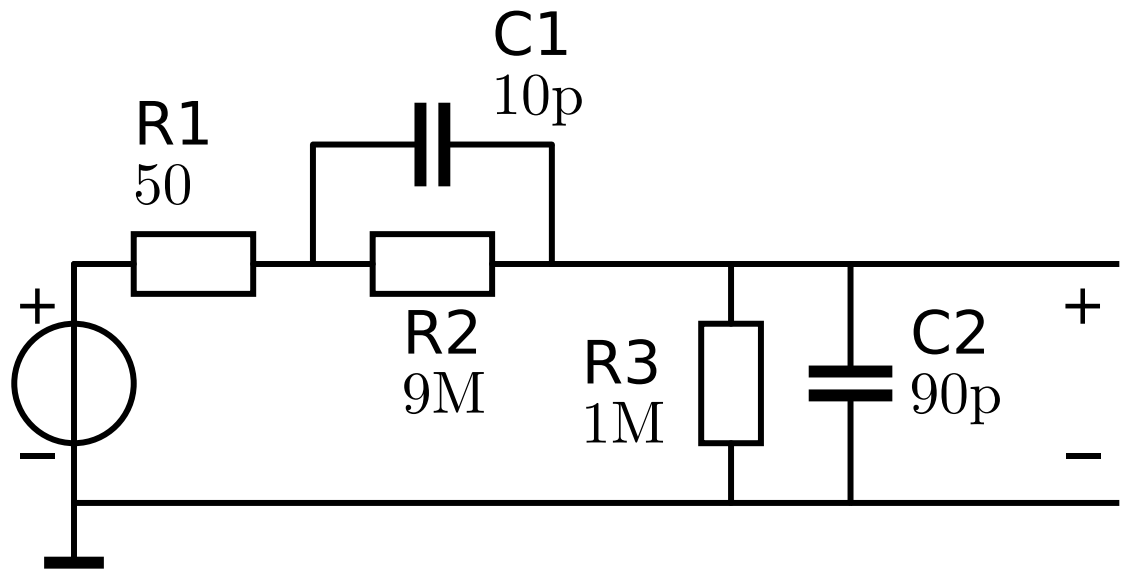
dominant pole



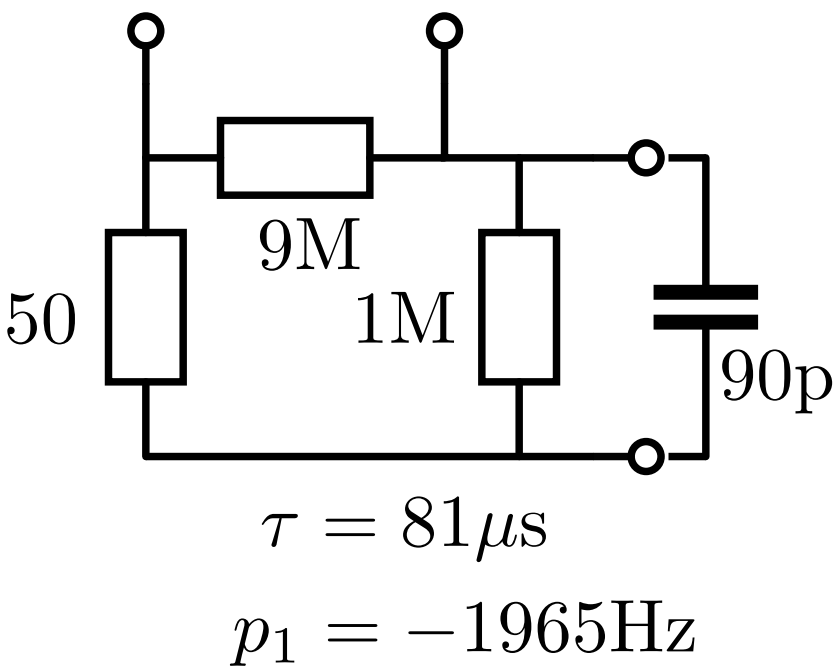
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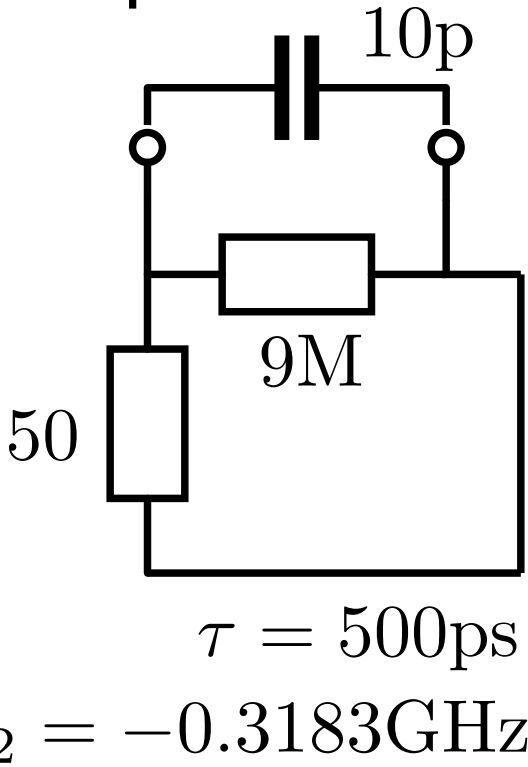
Oscilloscope probe circuit



dominant pole



non-dominant pole



# Rules

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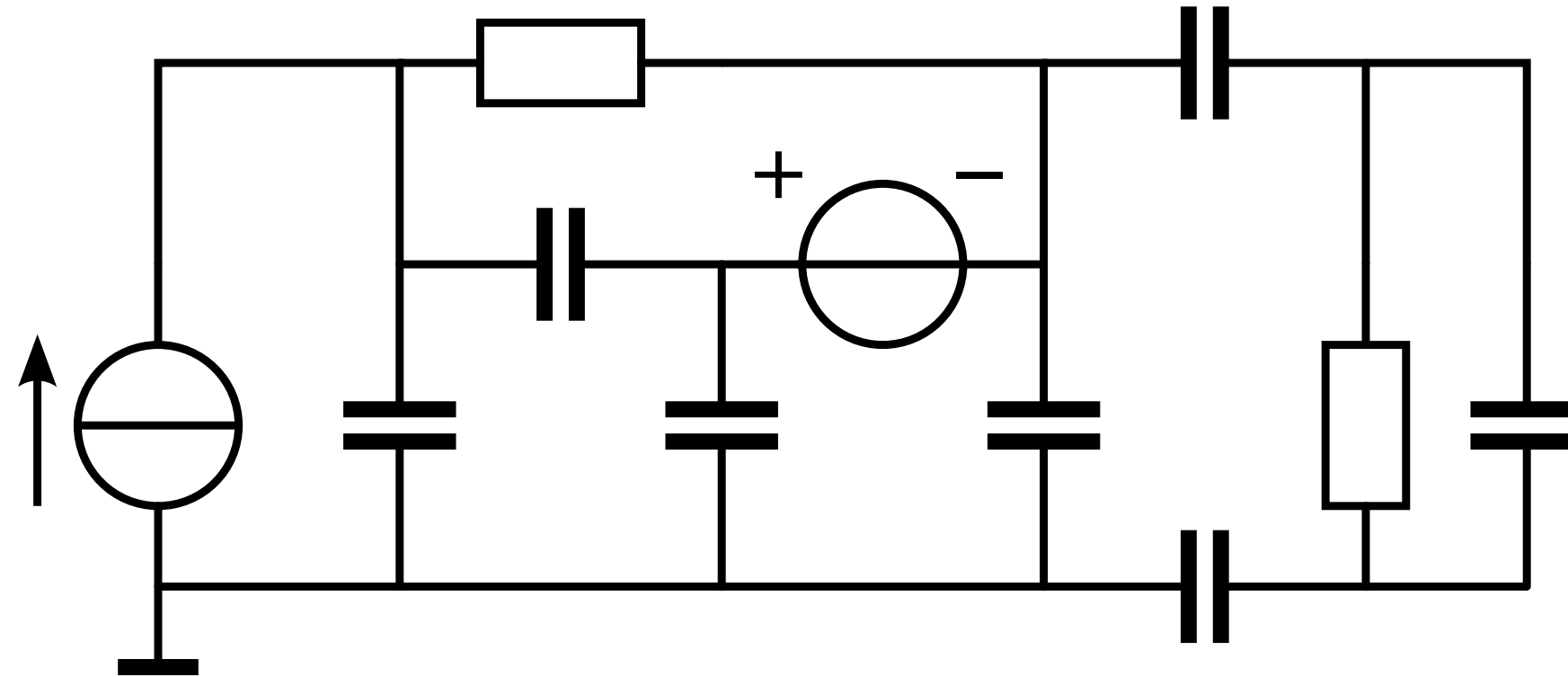
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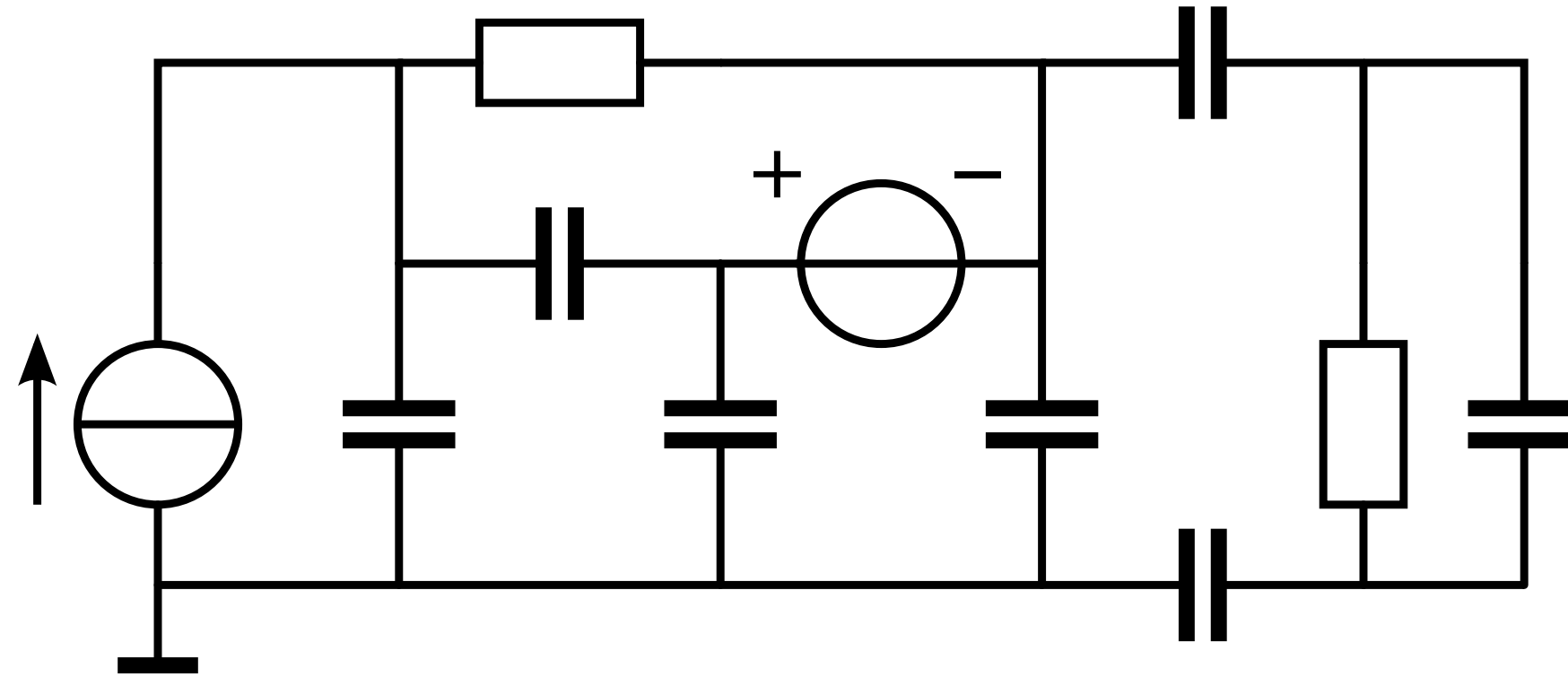
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# Example



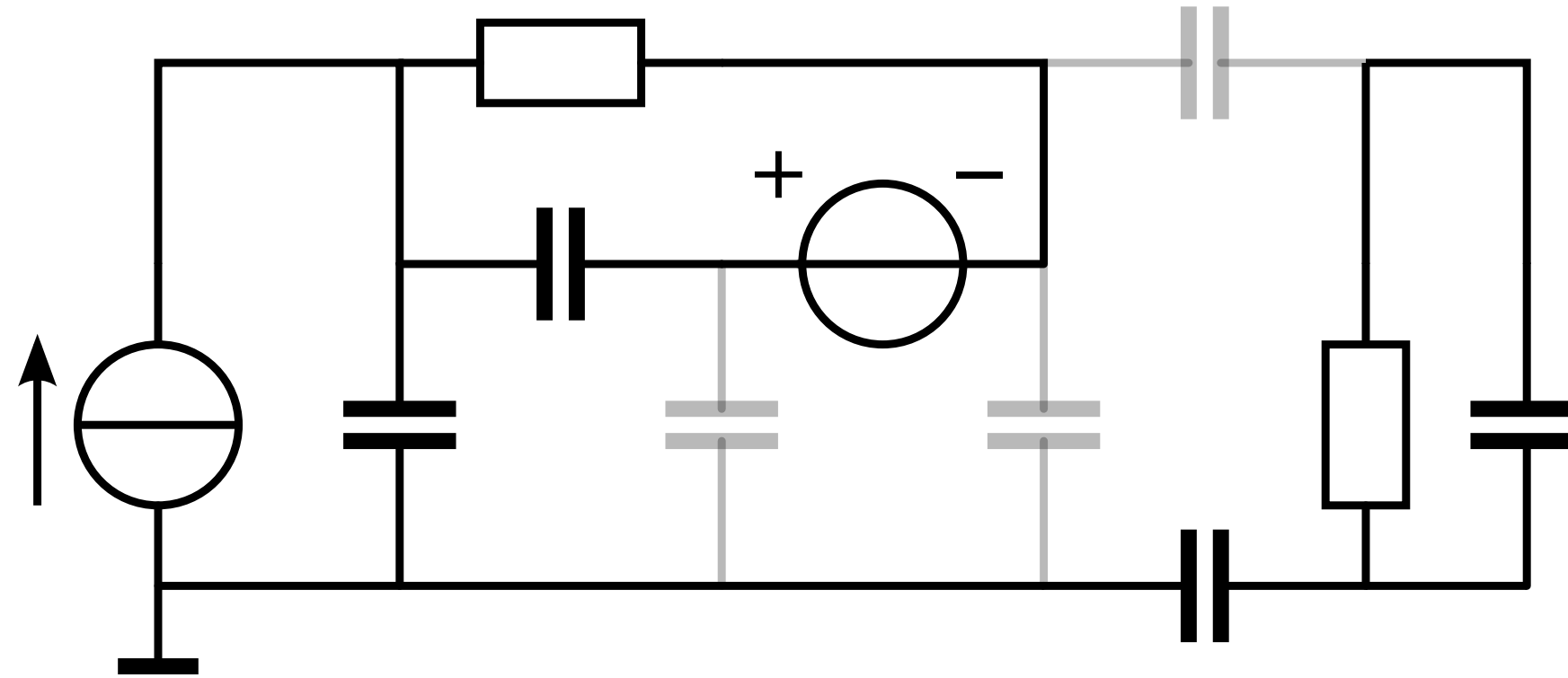
1. How many capacitors?
2. How many independent loops of capacitors and voltage sources?
3. How many poles?
4. How many independent cut sets of capacitors or of capacitors and current sources?
5. How many poles at  $s=0$ ?

# Example



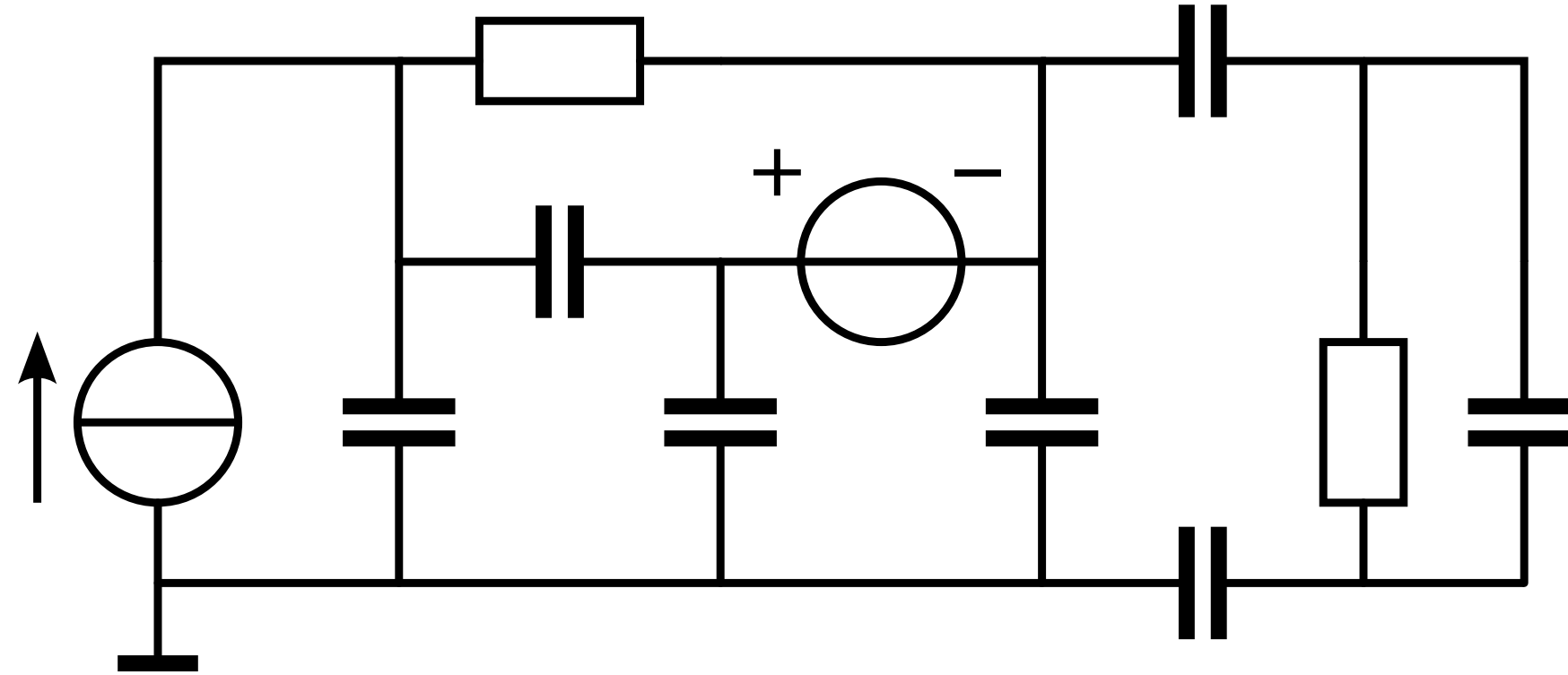
1. How many capacitors? 7

# Example



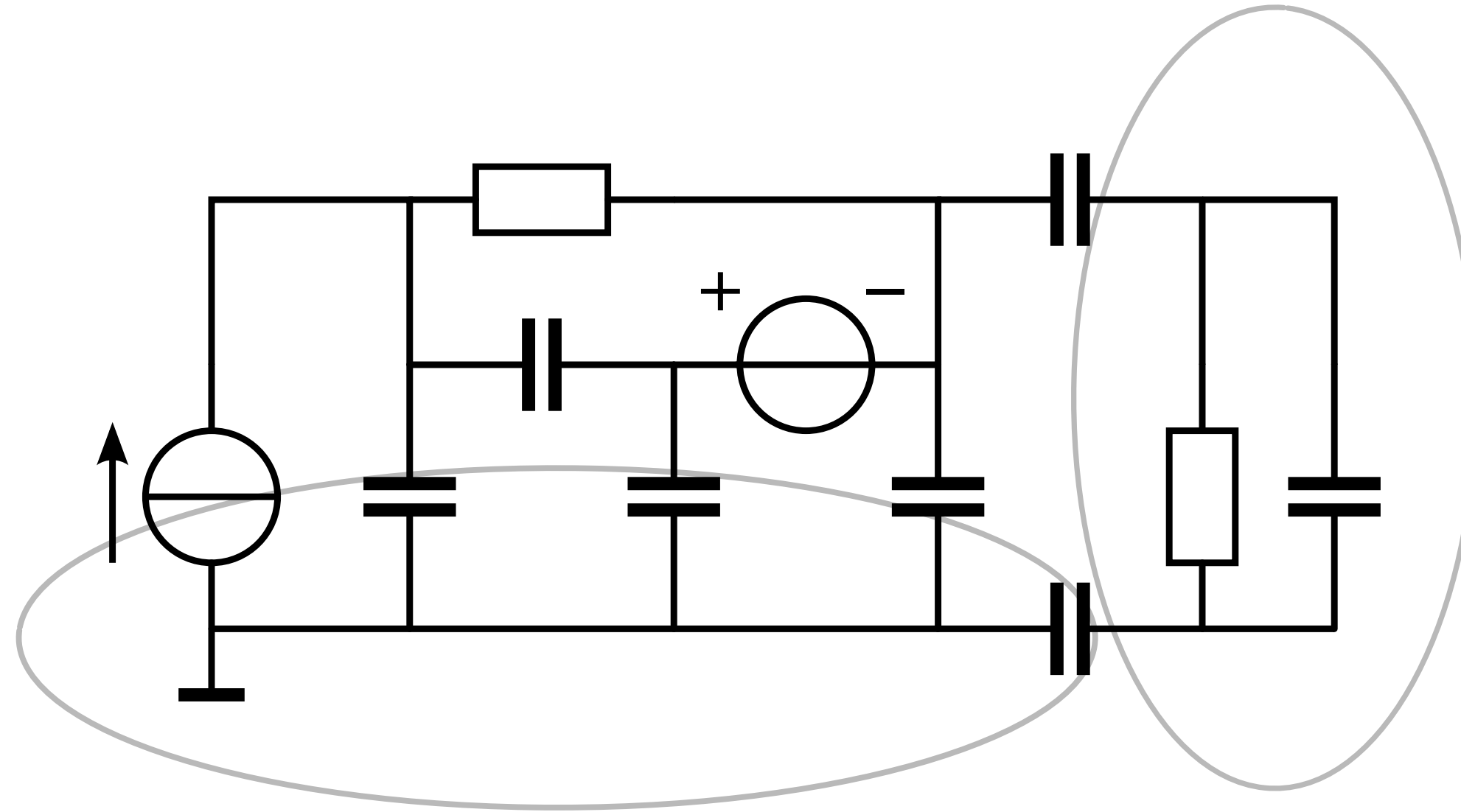
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# Example



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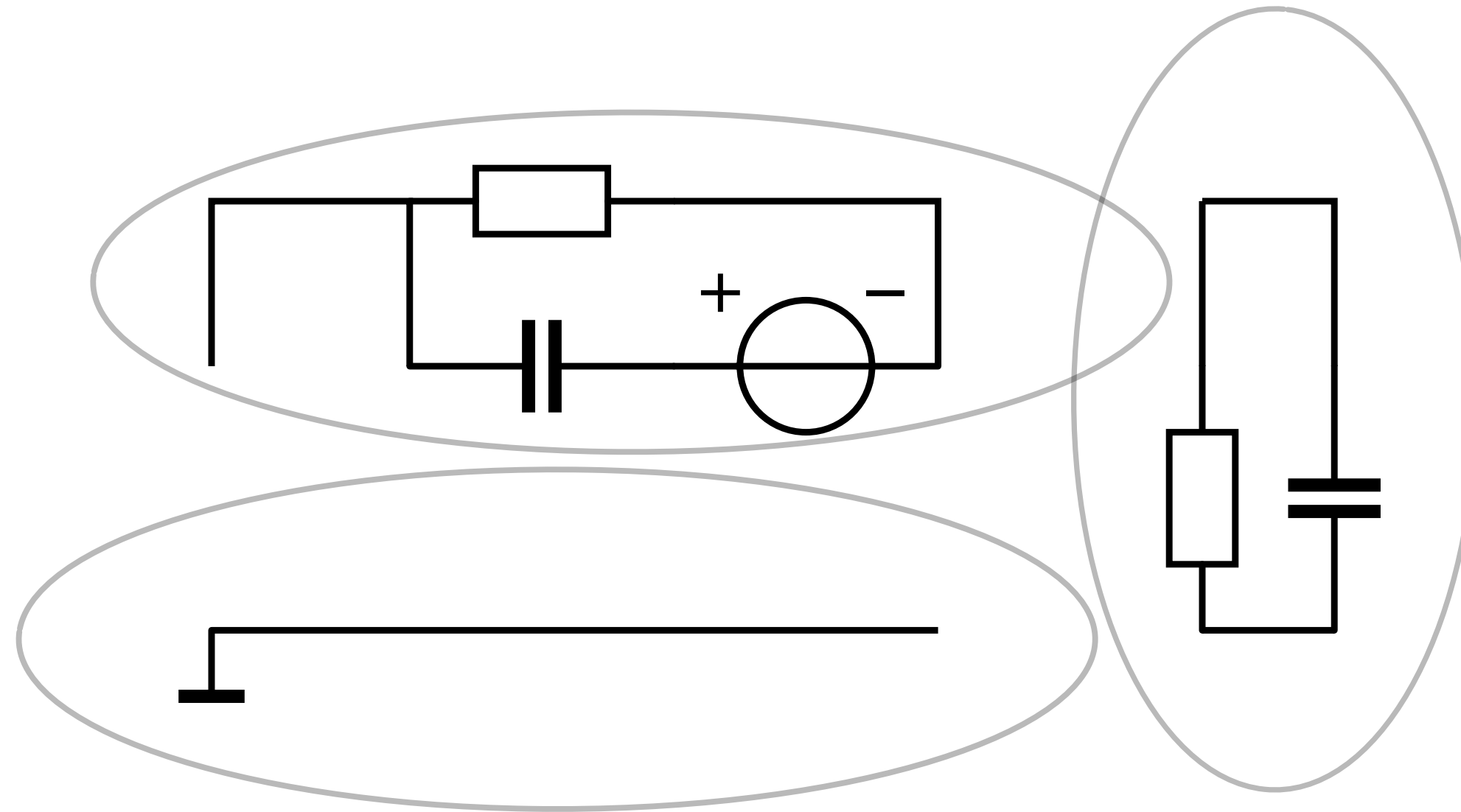
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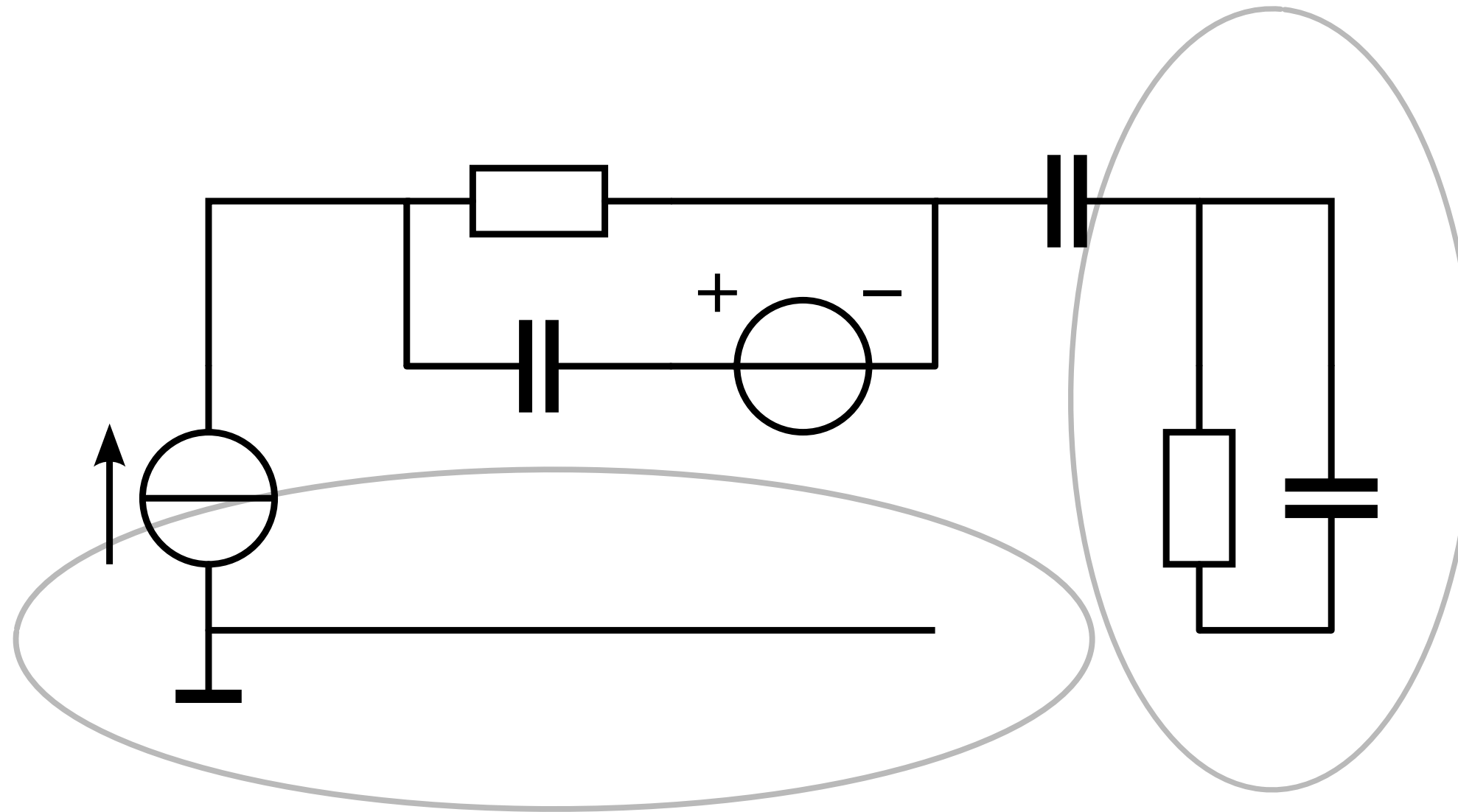


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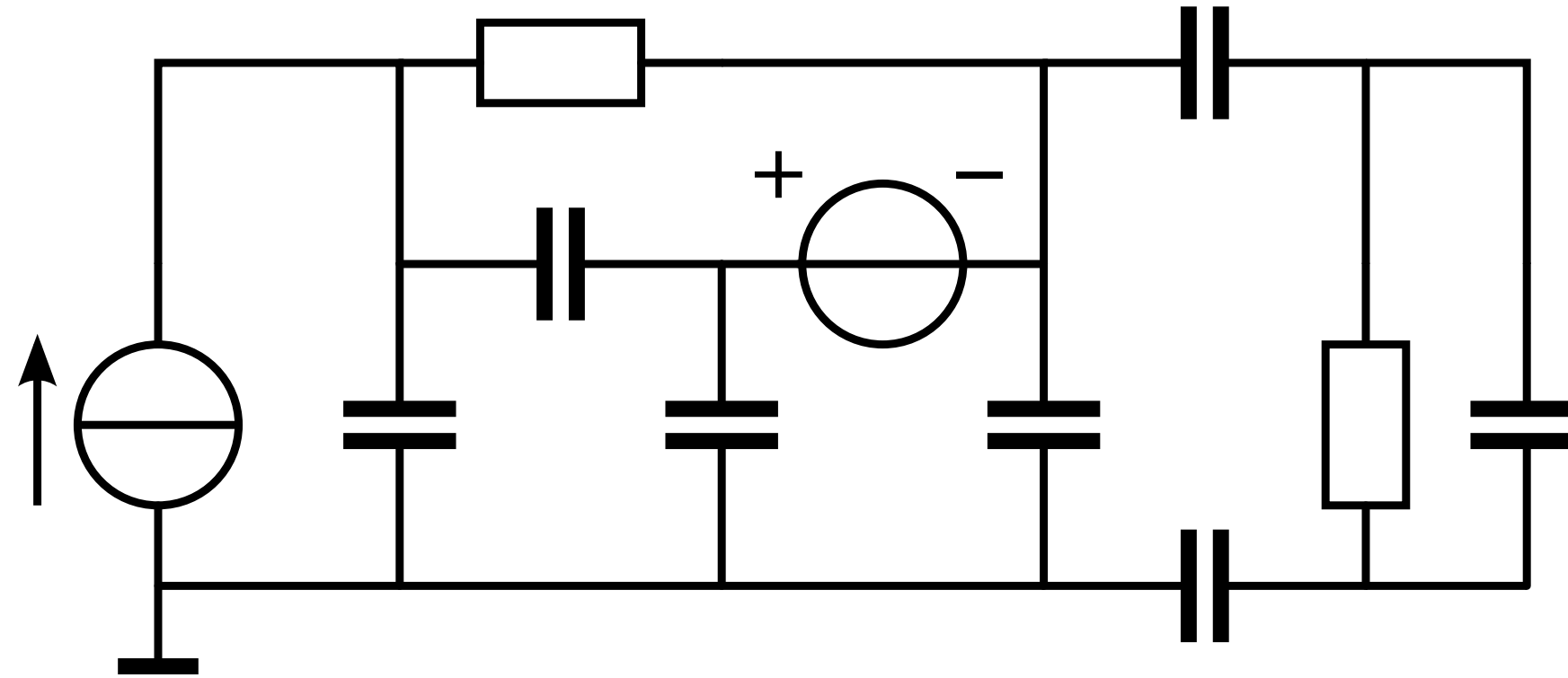
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# Zeros

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Zero transfer at the (complex) frequency of the zero

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Procedure for finding zeros

Short circuit in parallel with the signal path at complex frequency

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Transfer through multiple paths that cancel each other at complex frequency

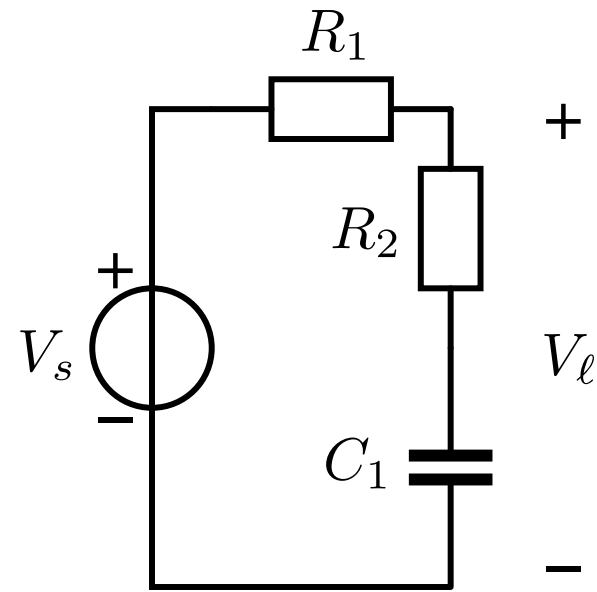


# Zeros

Short circuit in parallel with the signal path at complex frequency

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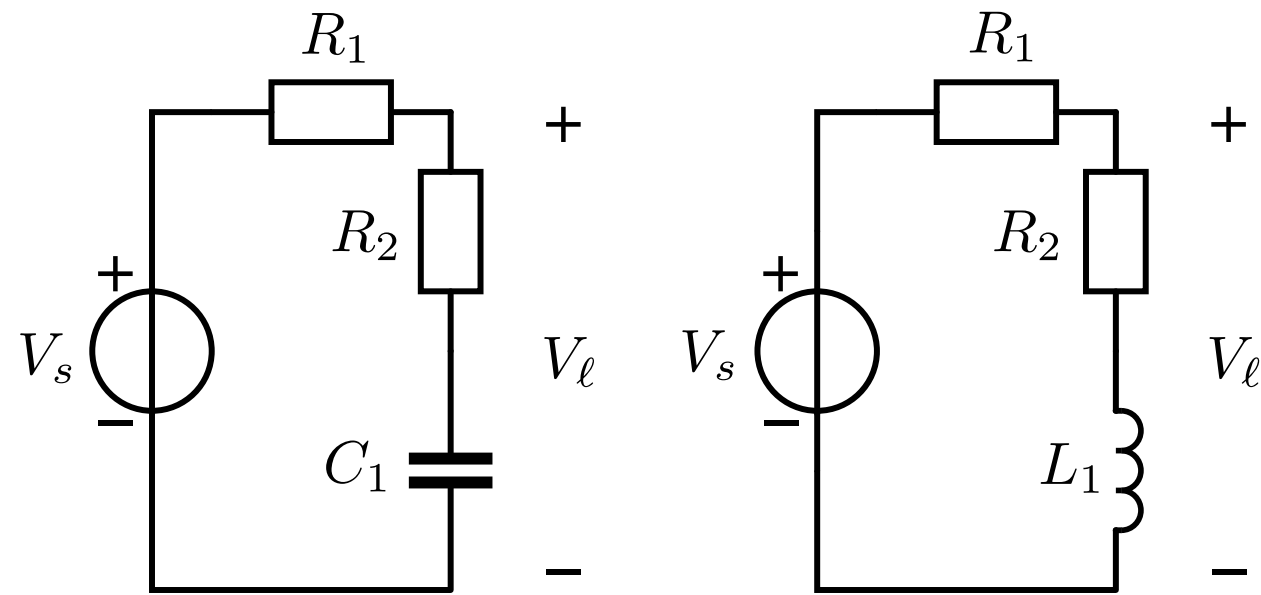
Short circuit in parallel with the signal path at complex frequency



$$R_2 + \frac{1}{sC_1} = 0$$

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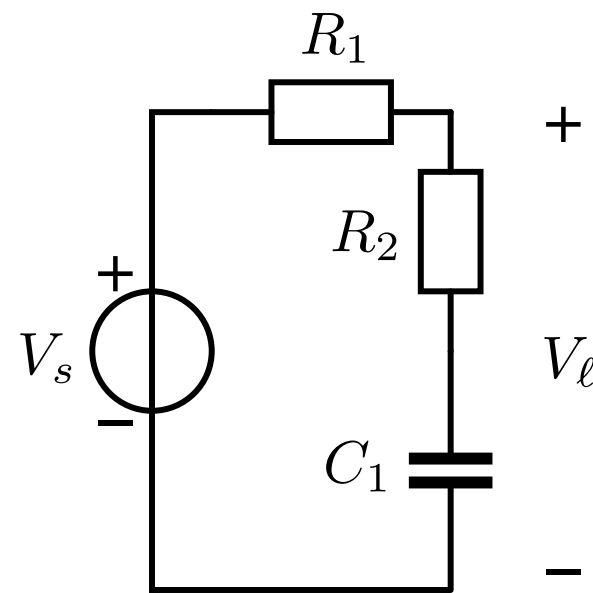


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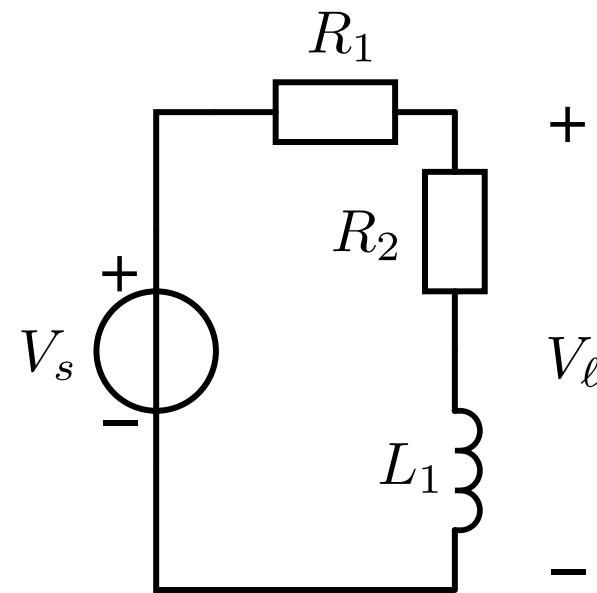
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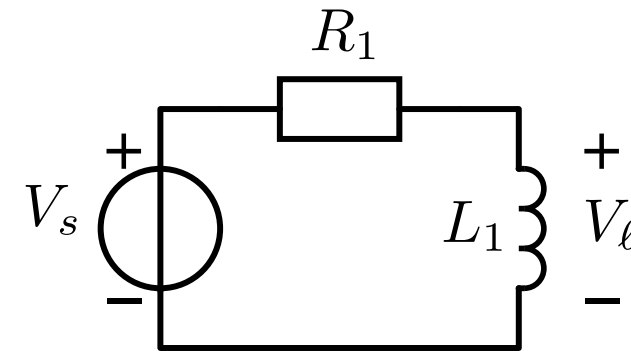
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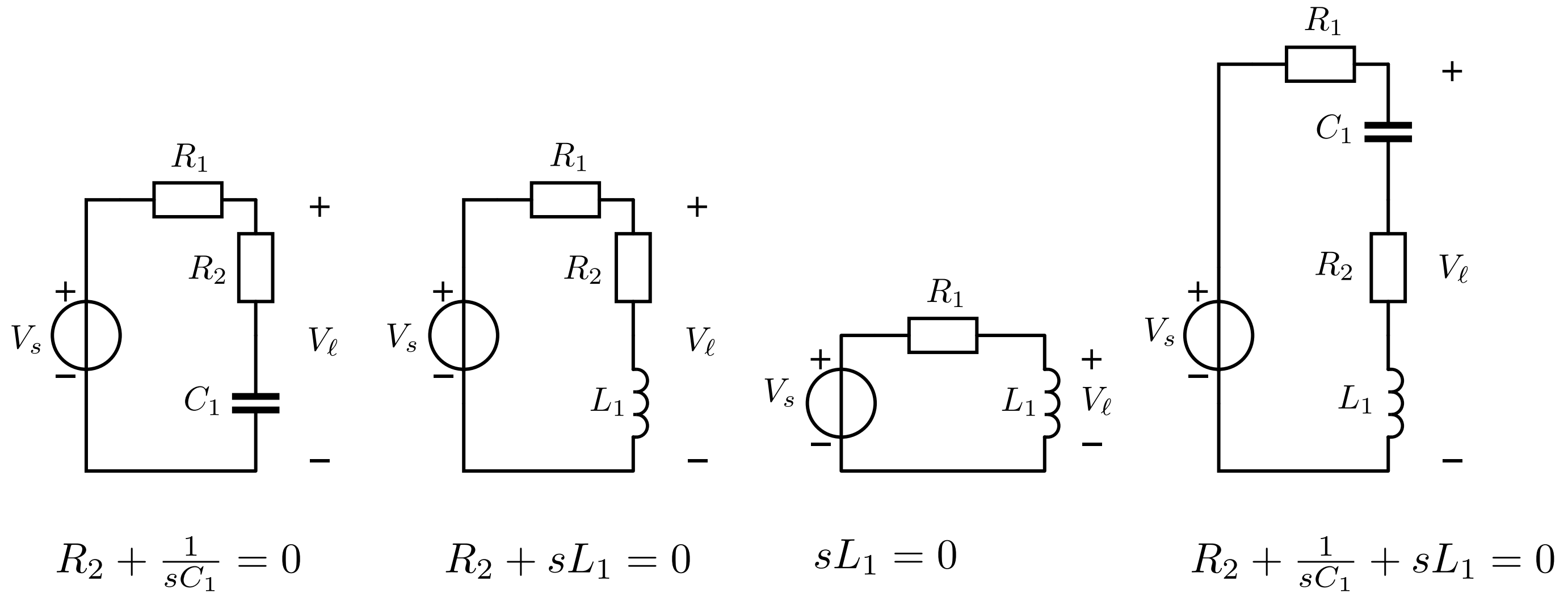
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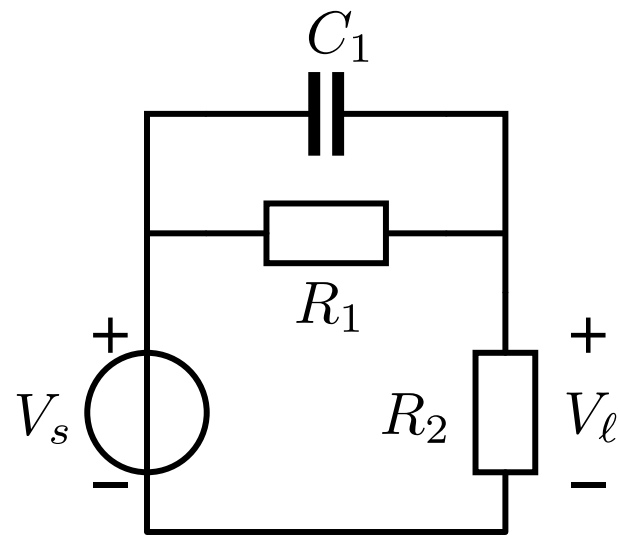


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Open circuit in series with the signal path at complex frequency

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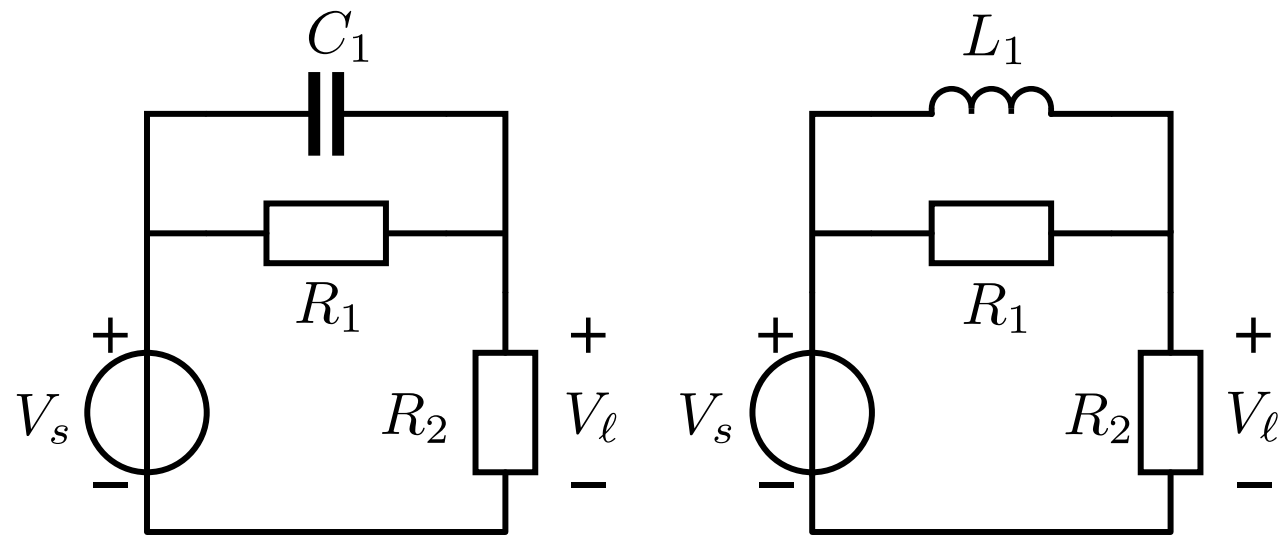
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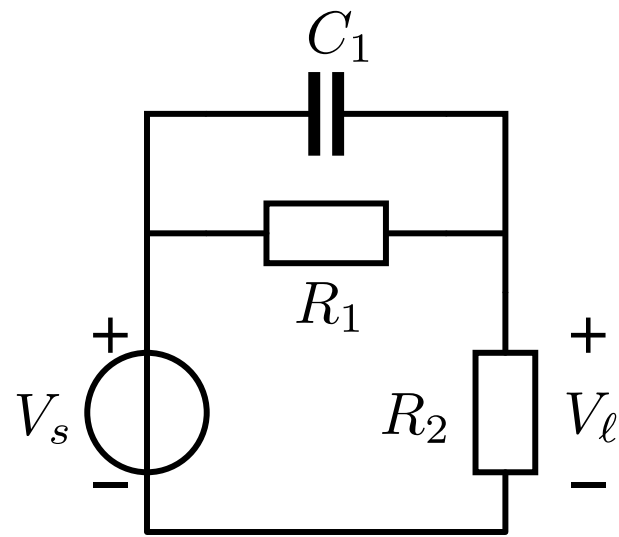
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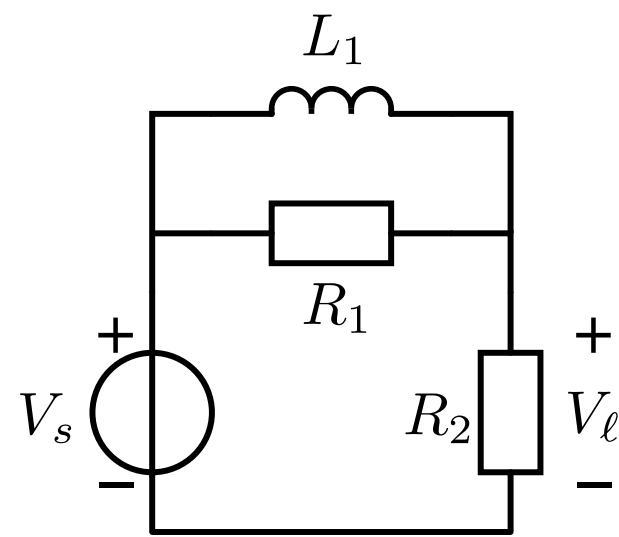


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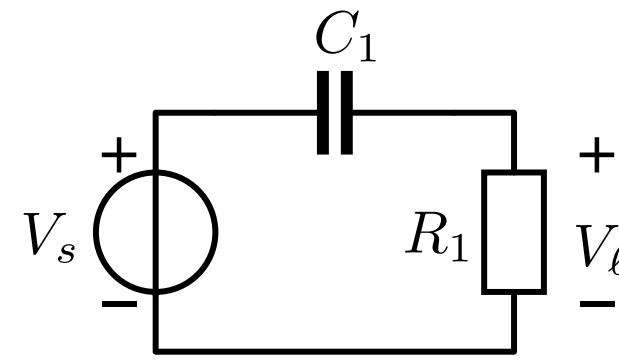
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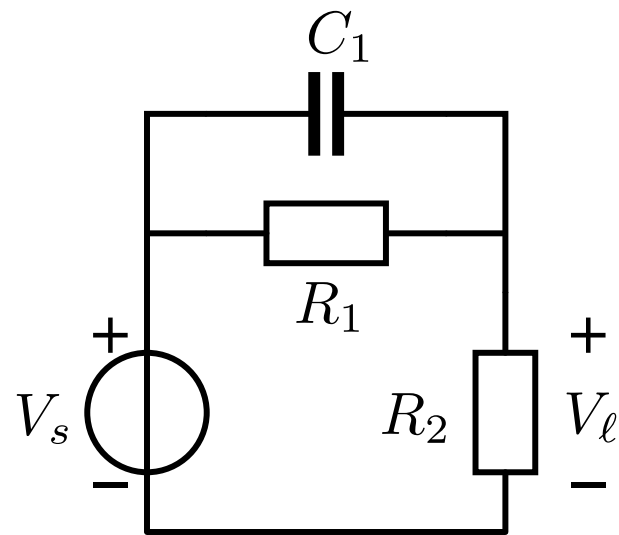
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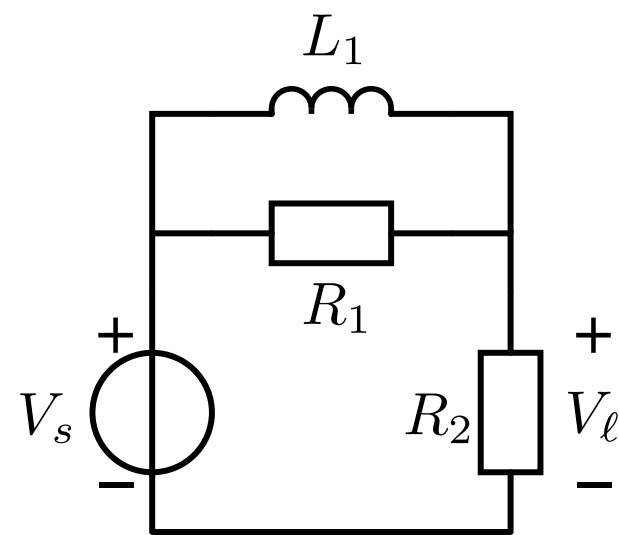
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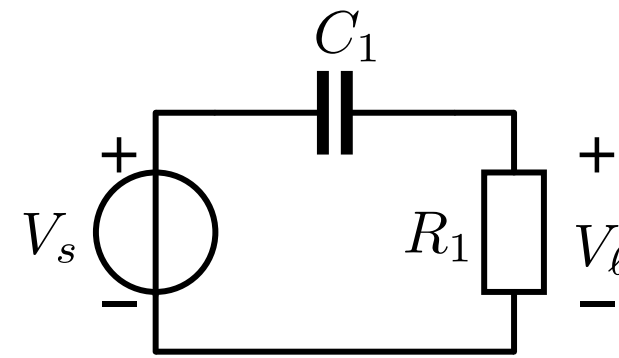
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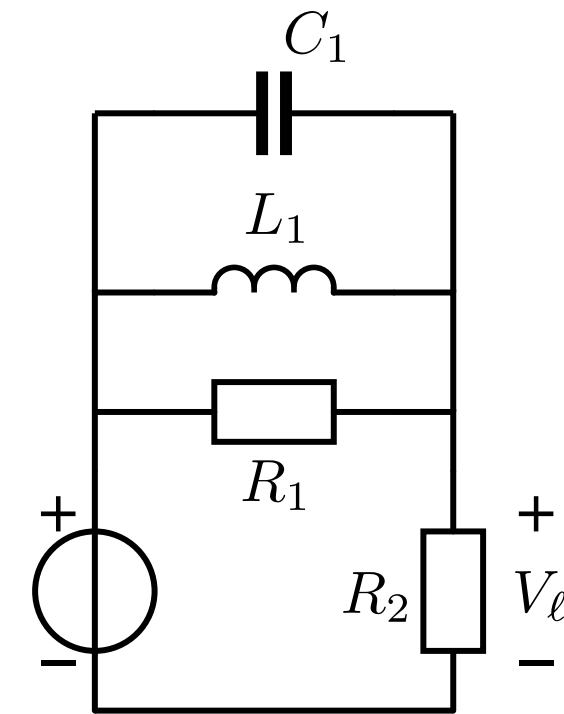
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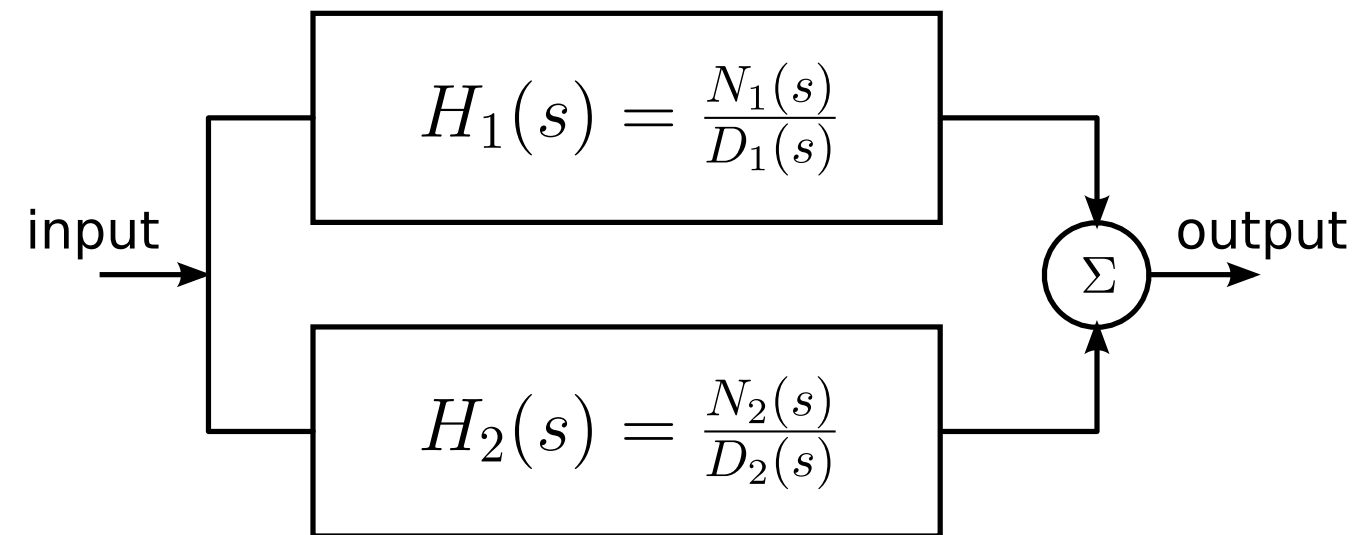
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# Zeros

Transfer through multiple paths that cancel each other at complex frequency



$$H_1(s) + H_2(s) = \frac{N_1(s)D_2(s) + N_2(s)D_1(s)}{D_1(s)D_2(s)}$$