Structured Electronic Design
Analysis and budgeting of biasing errors

Modeling and analysis of biasing errors

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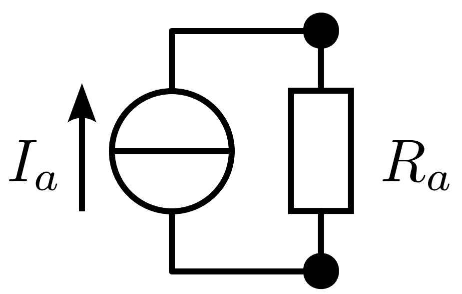

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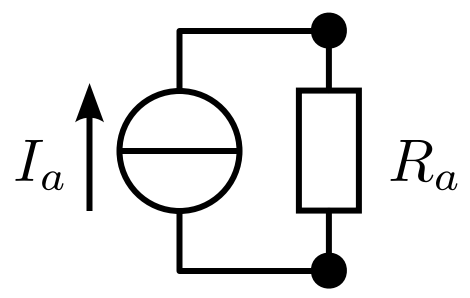

mean value: $\overline{I_a}$ $\overline{R_a}$

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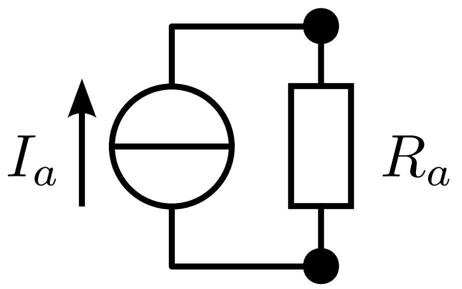

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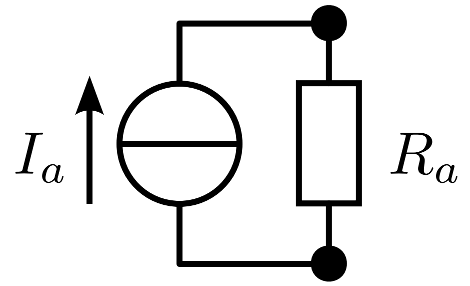

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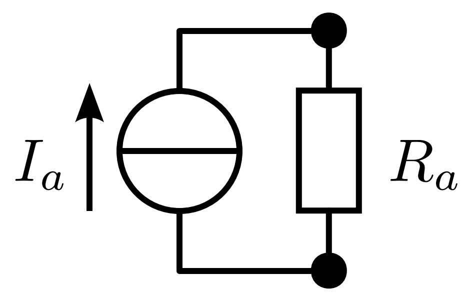

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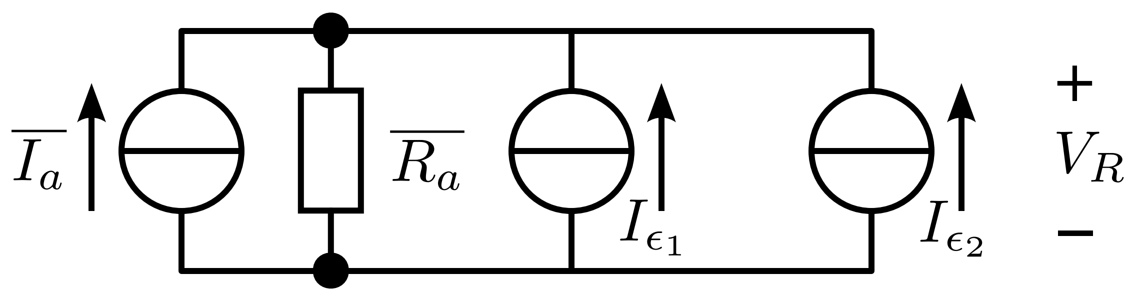

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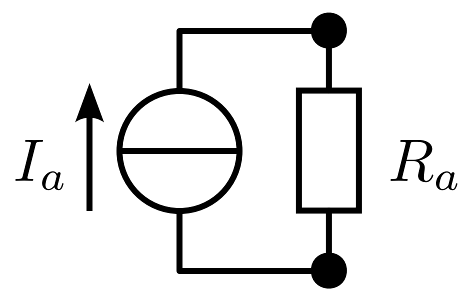

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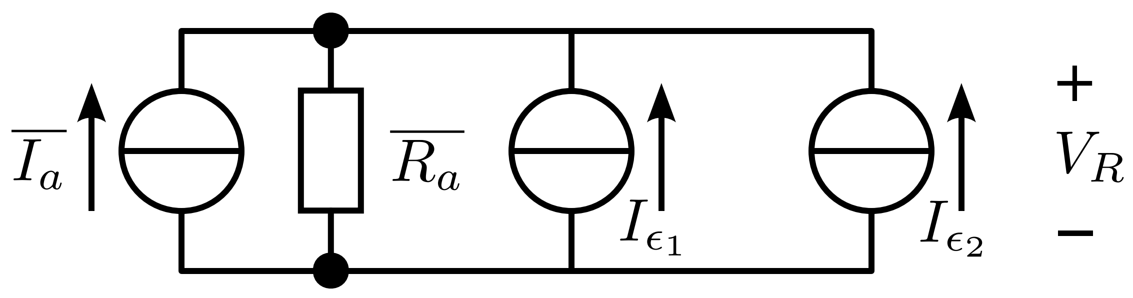

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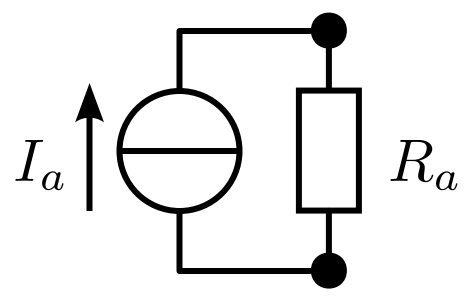

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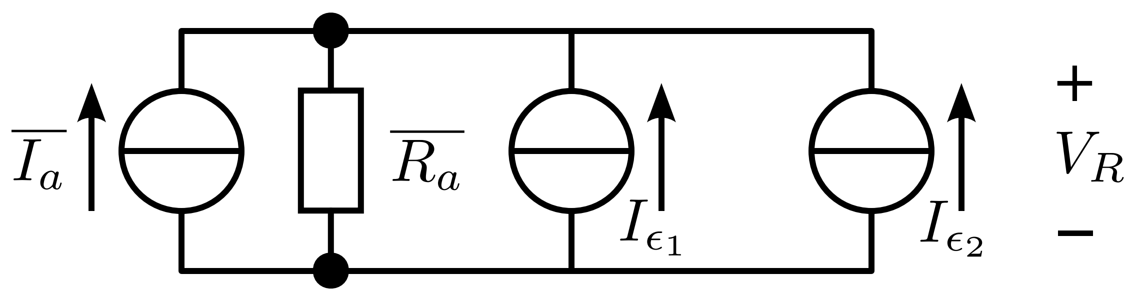

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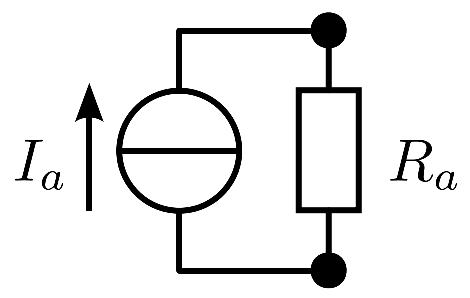

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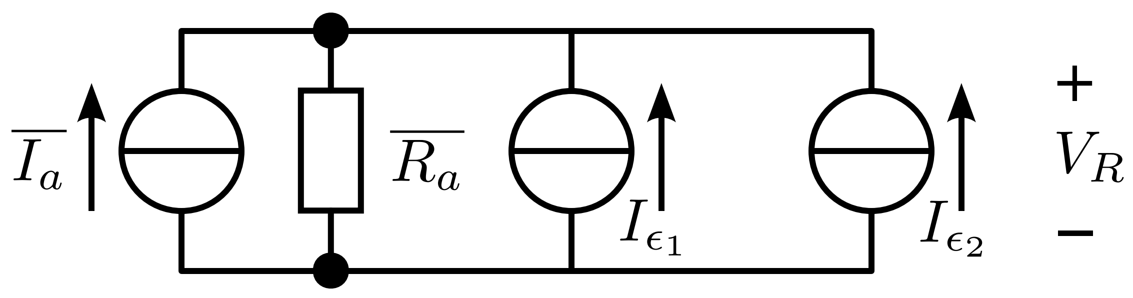

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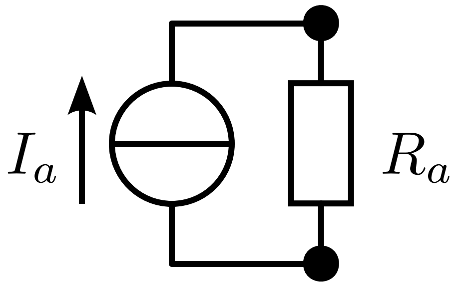

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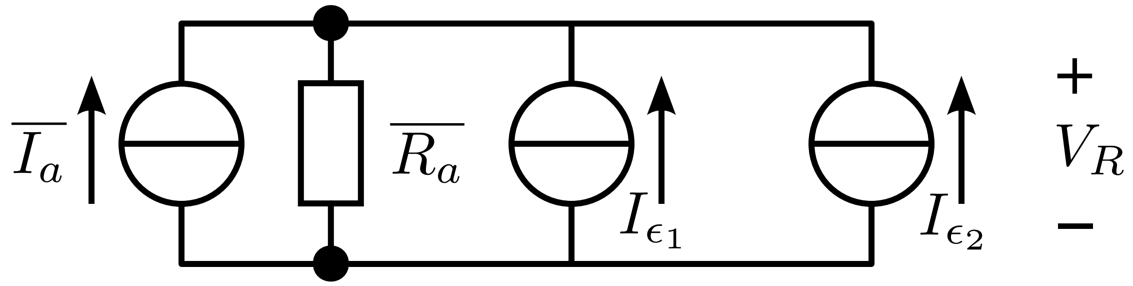

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Calculation of the variance of the voltage across the resistor:

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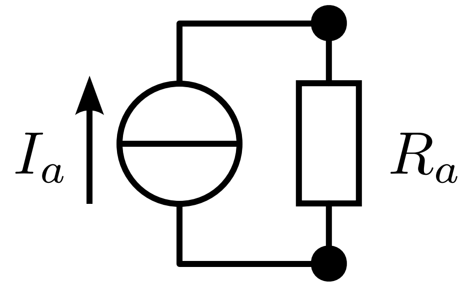

mean value: $\overline{I_a}$ $\overline{R_a}$

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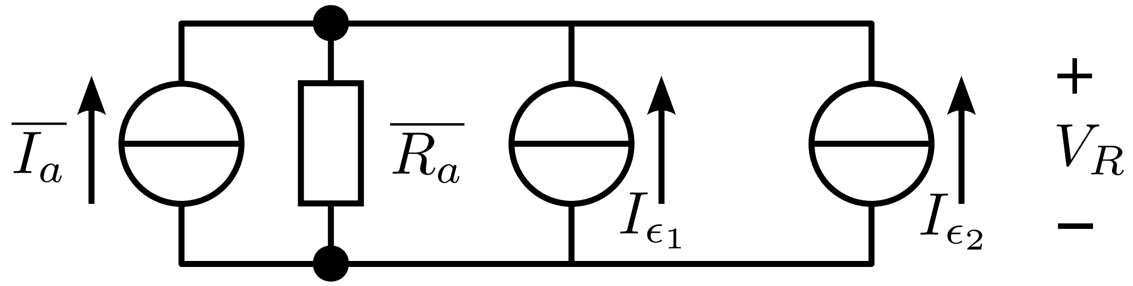

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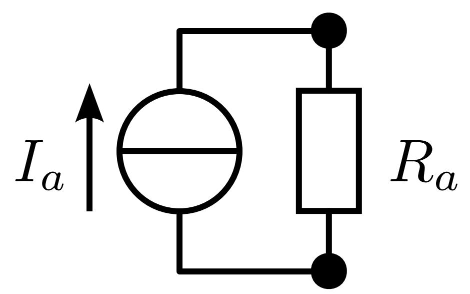

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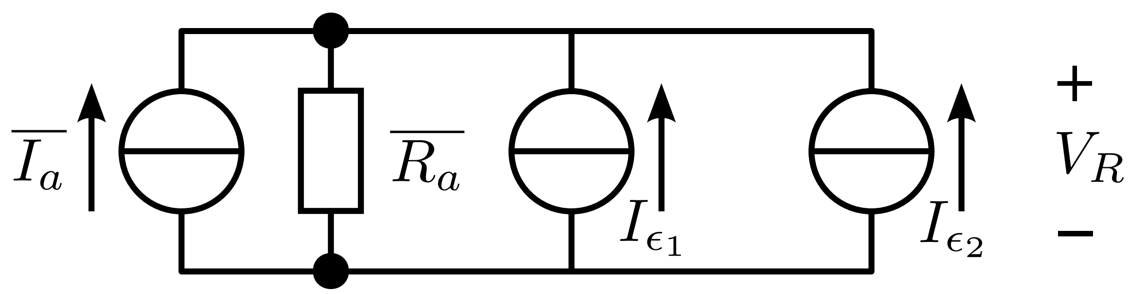

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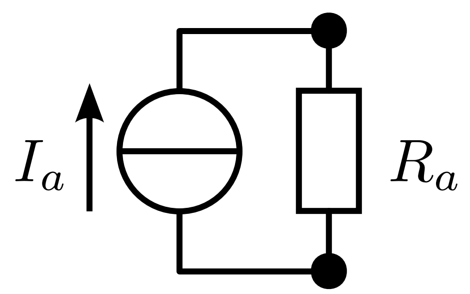

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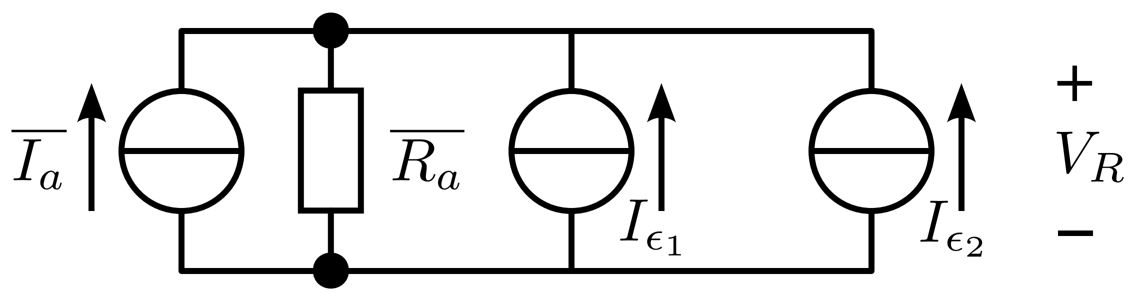

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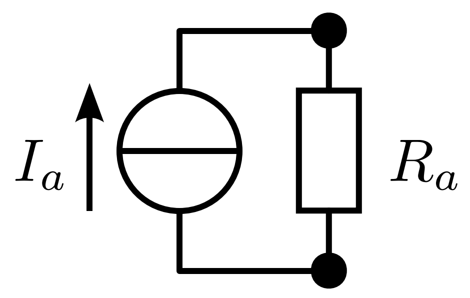

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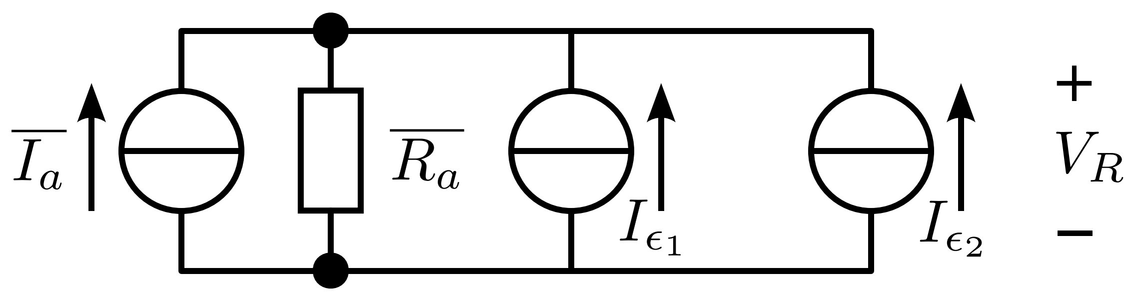

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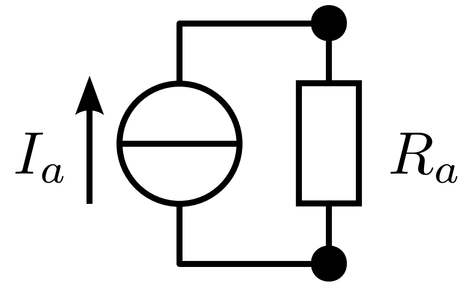

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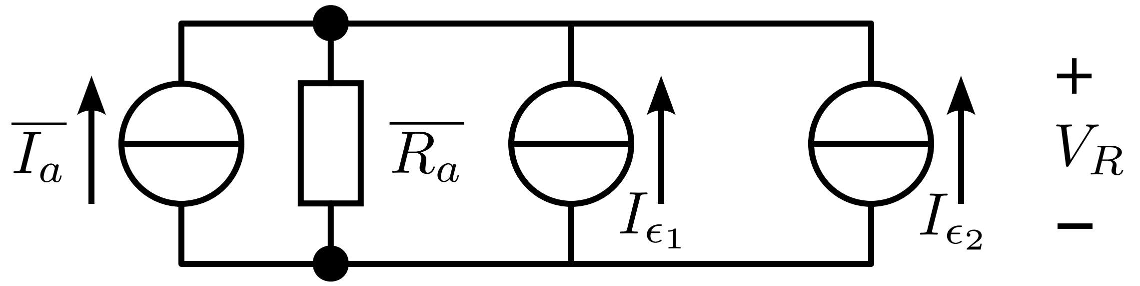

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Influence of supply and resistor tolerances

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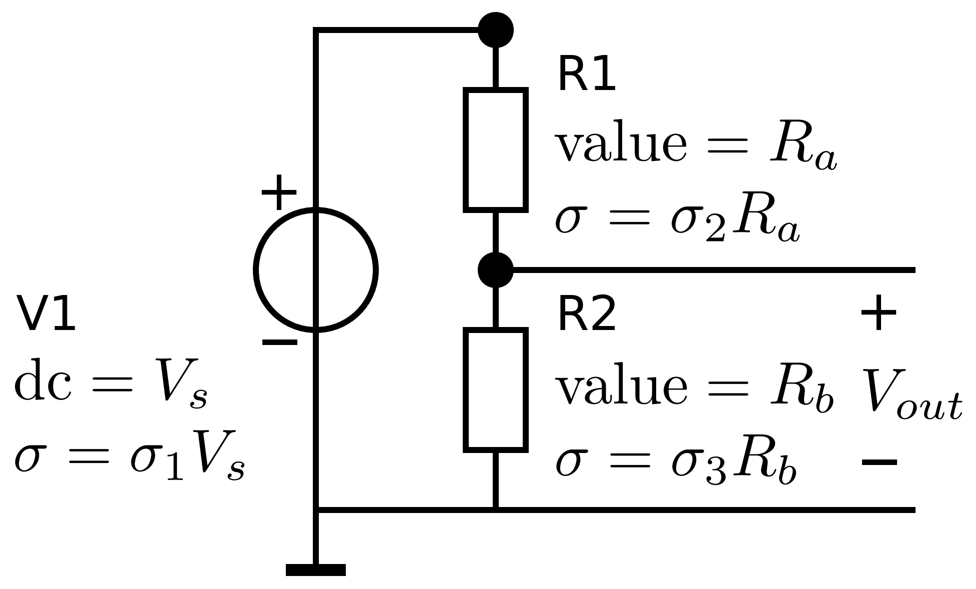
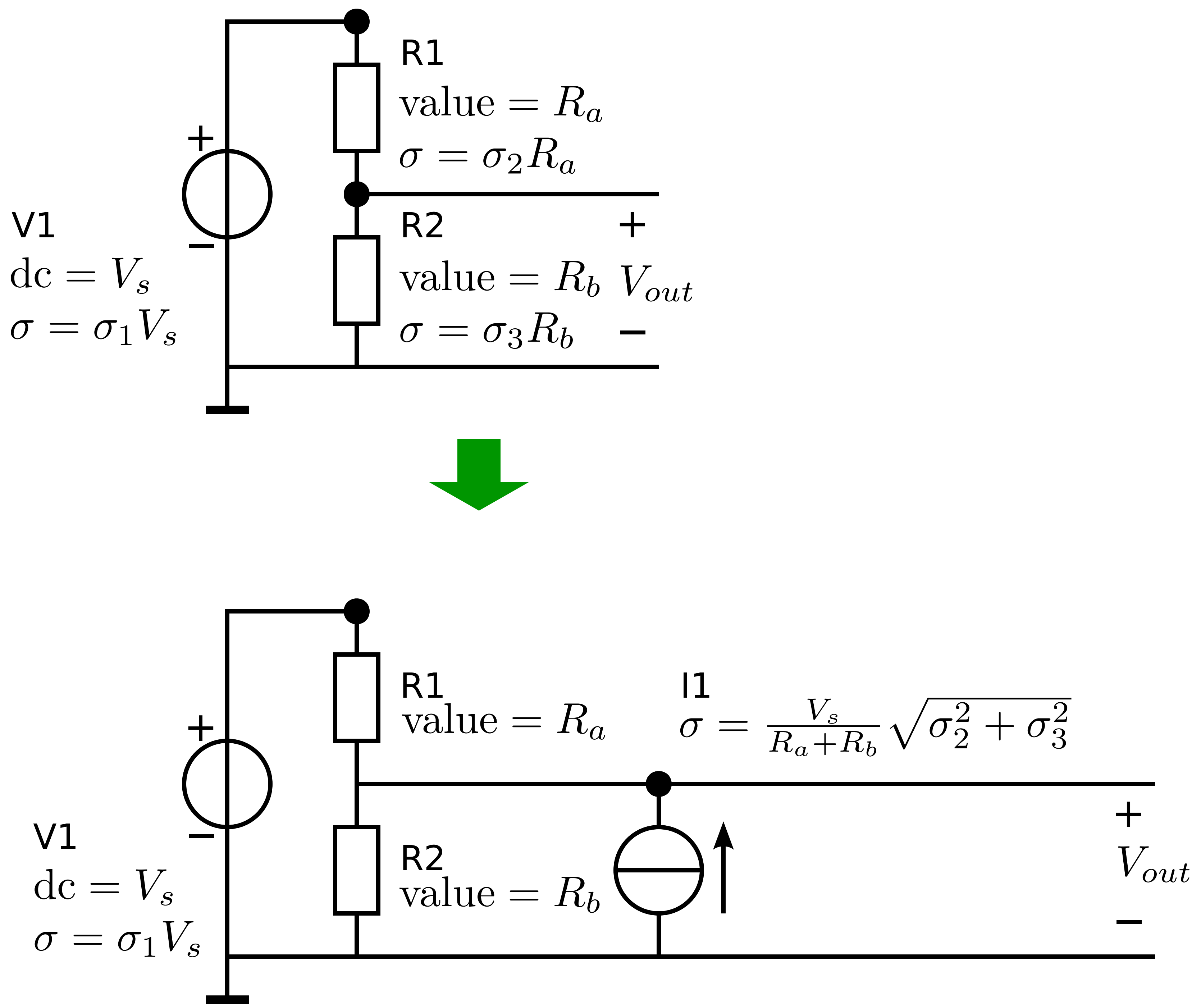
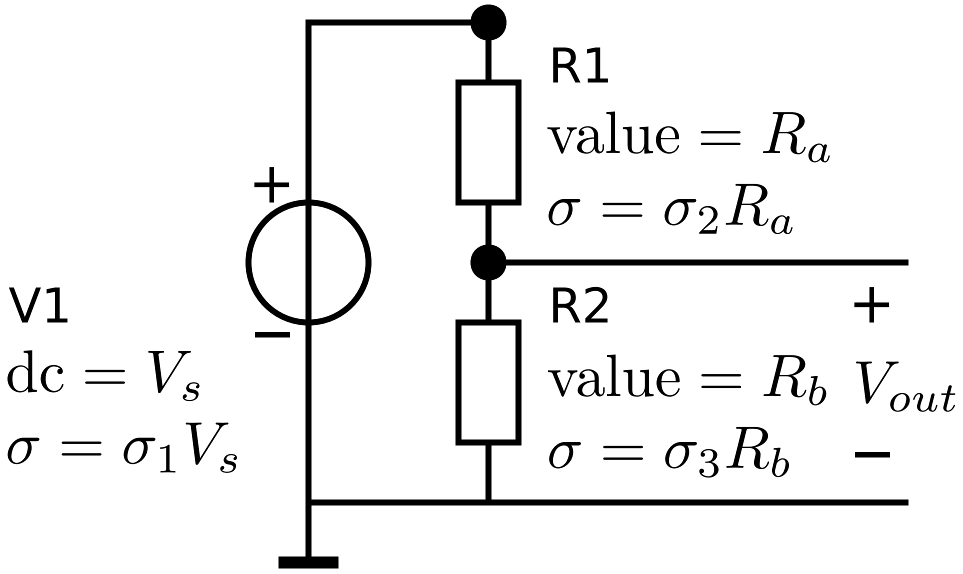
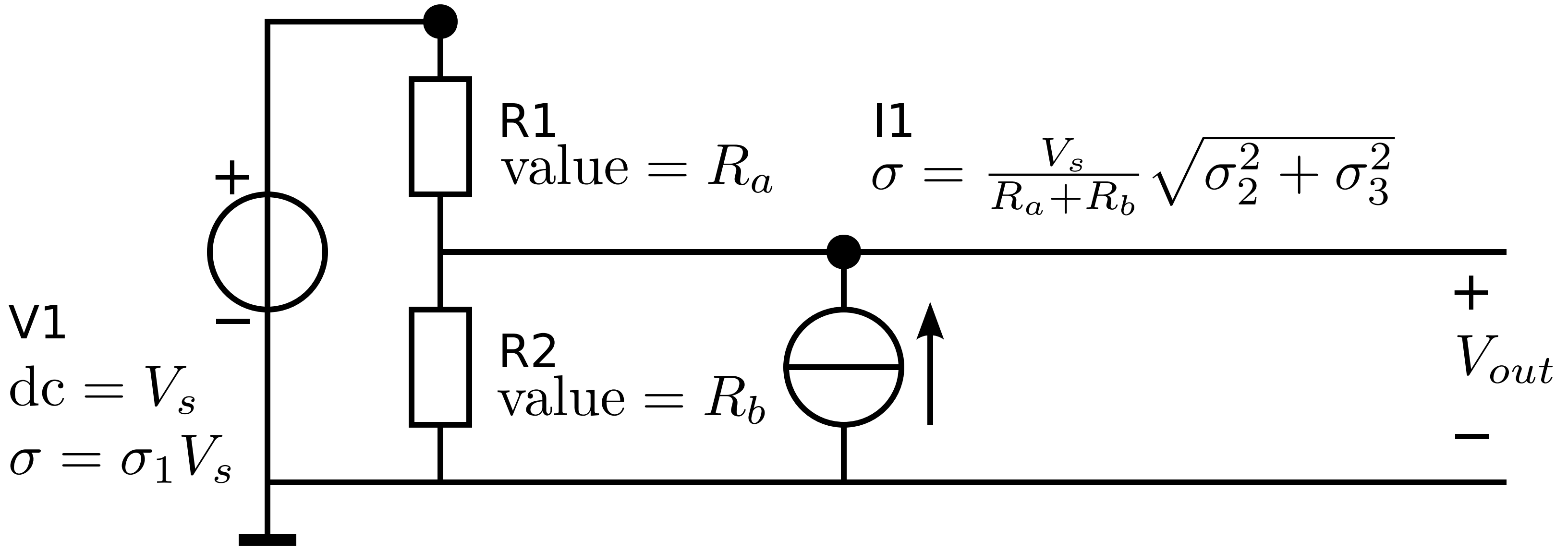


Influence of supply and resistor tolerances



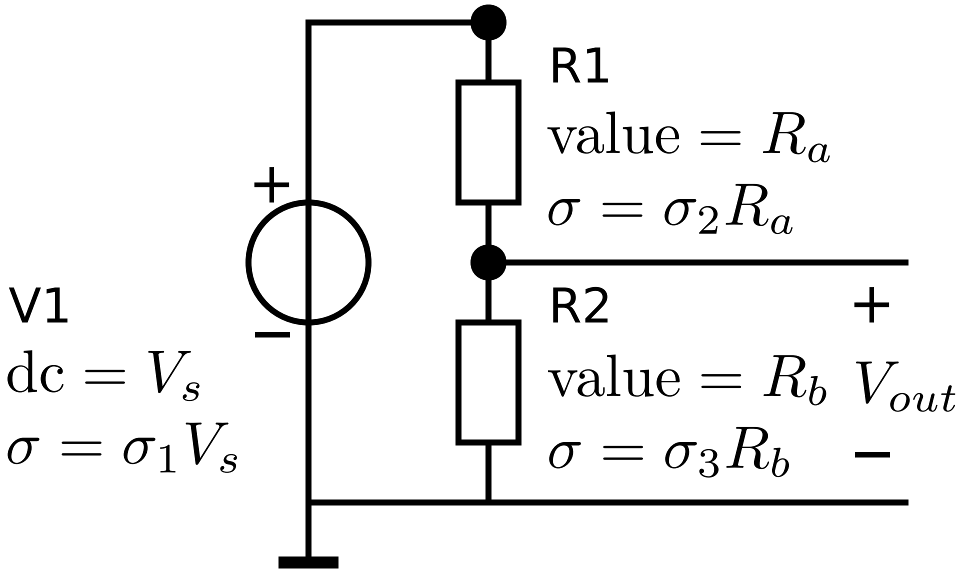
Influence of supply and resistor tolerances

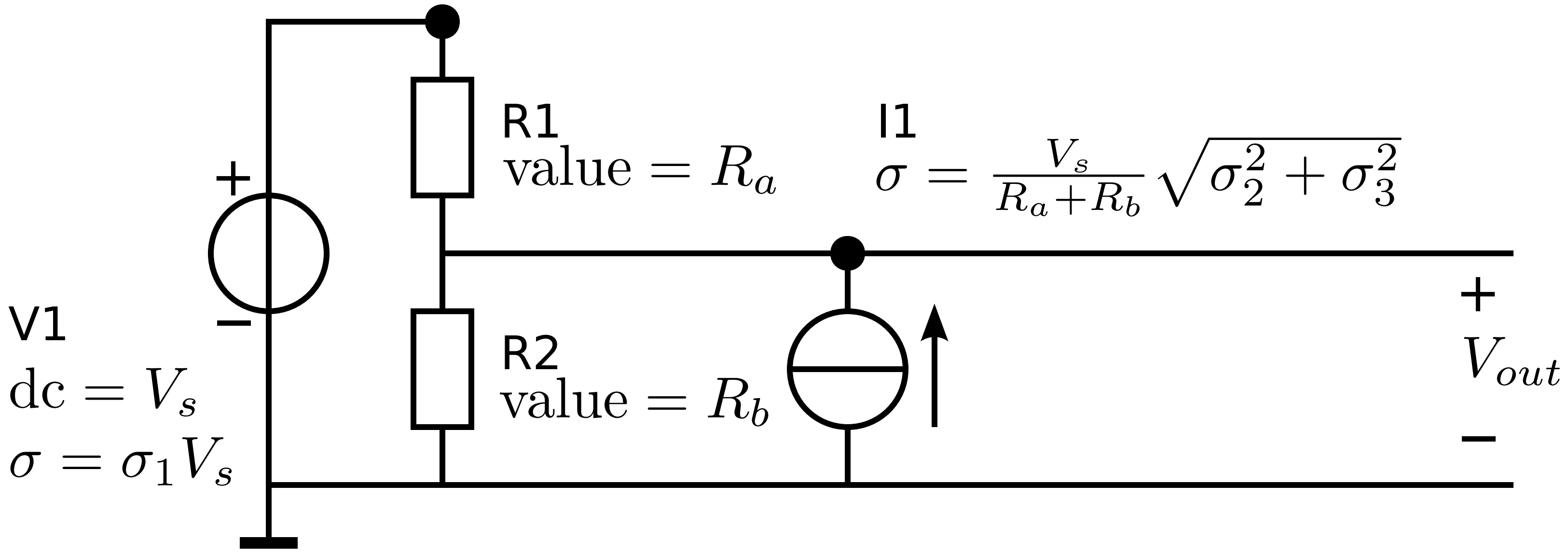




$$\overline{V_{out}} = V_s \frac{R_b}{R_a+R_b}$$

Influence of supply and resistor tolerances

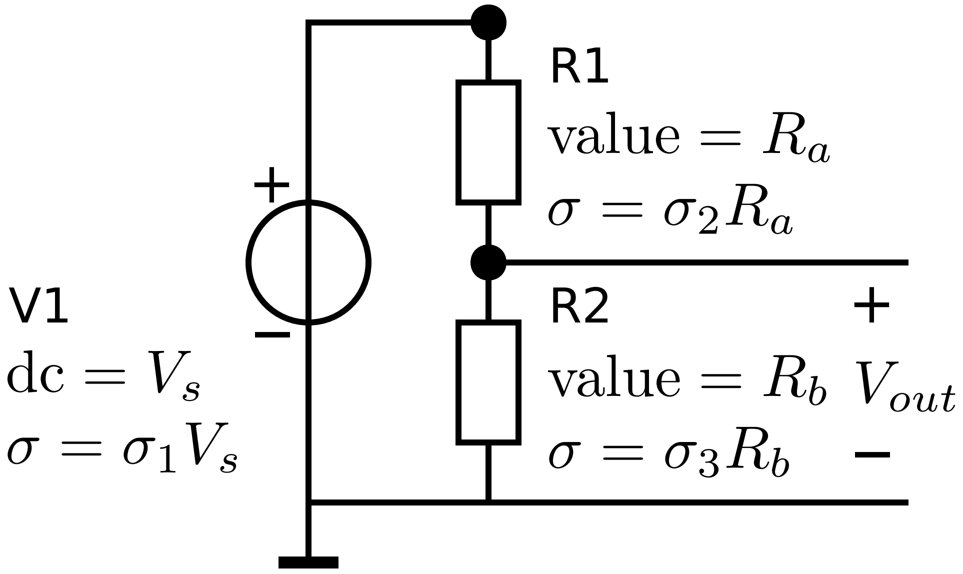


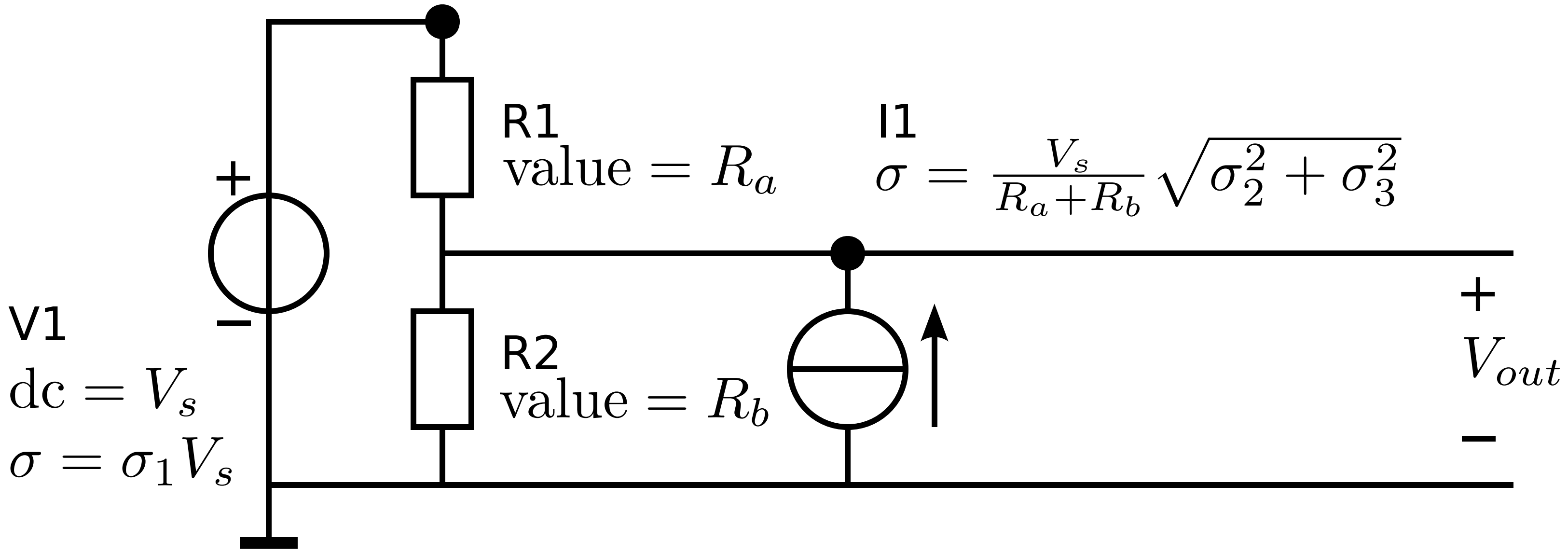


$$\overline{V_{out}} = V_s \frac{R_b}{R_a+R_b}$$

$$\frac{\sigma_{V_{out}}}{\overline{V_{out}}} = \sqrt{\left(\frac{R_a}{R_a+R_b}\right)^2 (\sigma_2^2+\sigma_3^2) + \sigma_1^2}$$

Influence of supply and resistor tolerances





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Influence of controller biasing errors

Influence of controller biasing errors

SLiCAP model of ideal
OpAmp with bias errors

Influence of controller biasing errors

SLiCAP model of ideal
OpAmp with bias errors

- Correlated bias currents

Influence of controller biasing errors

SLiCAP model of ideal
OpAmp with bias errors

- Correlated bias currents
- Uncorrelated offsets

Influence of controller biasing errors

SLiCAP model of ideal OpAmp with bias errors

- Correlated bias currents
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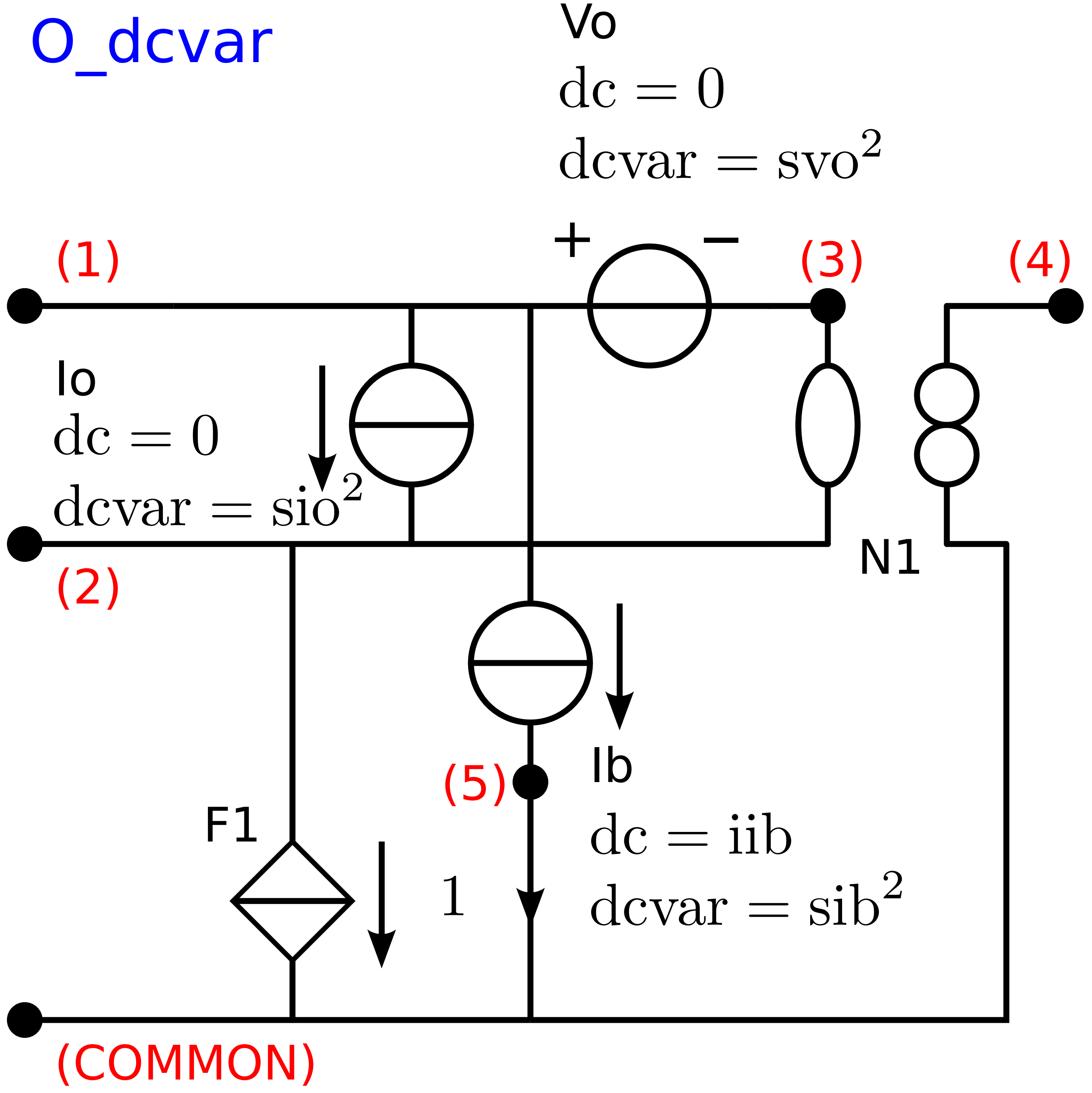
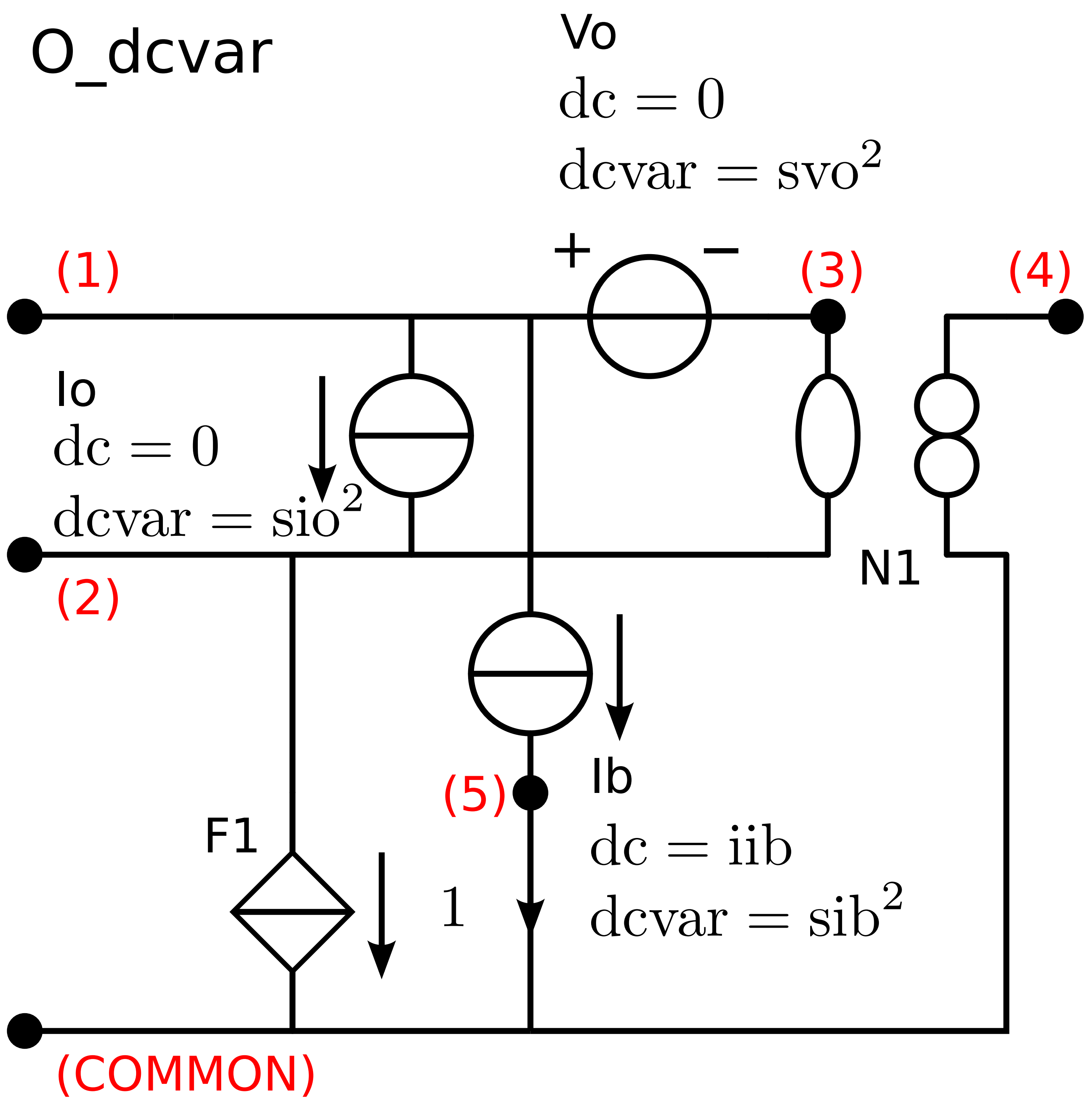


Influence of controller biasing errors

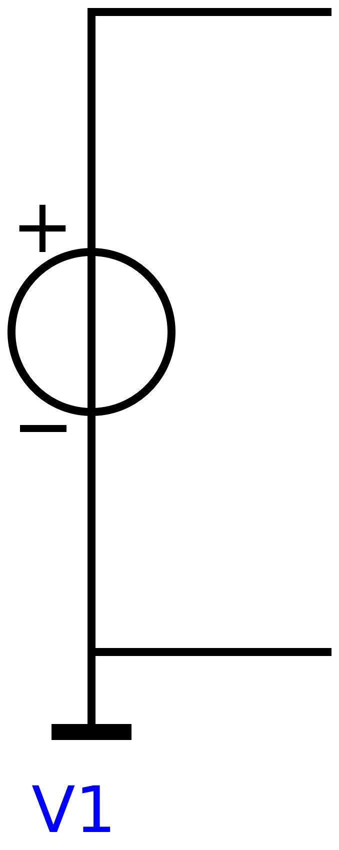
SLiCAP model of ideal OpAmp with bias errors

- Correlated bias currents
- Uncorrelated offsets



Total bias errors

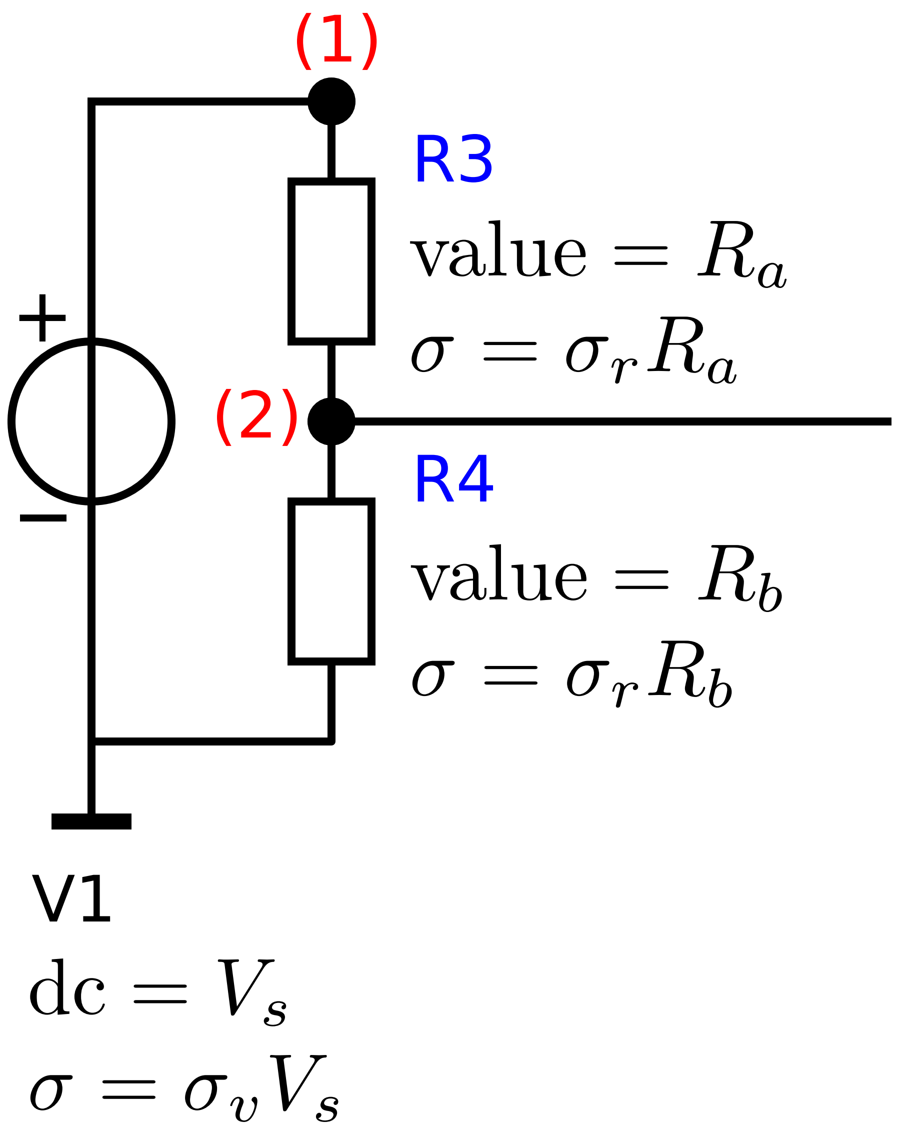
Total bias errors



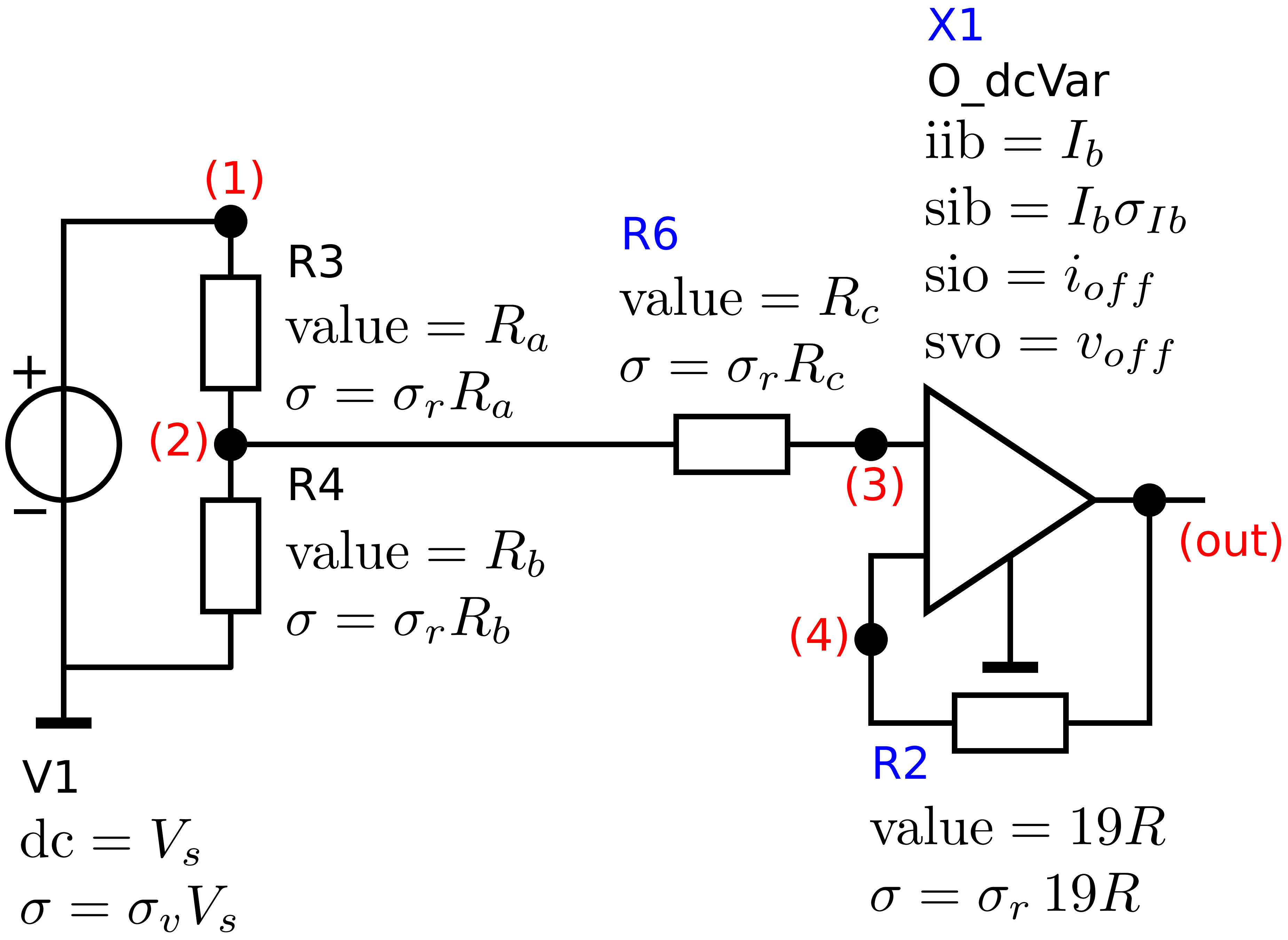
$\mathrm{dc} = V_s$

$\sigma = \sigma_v V_s$

Total bias errors

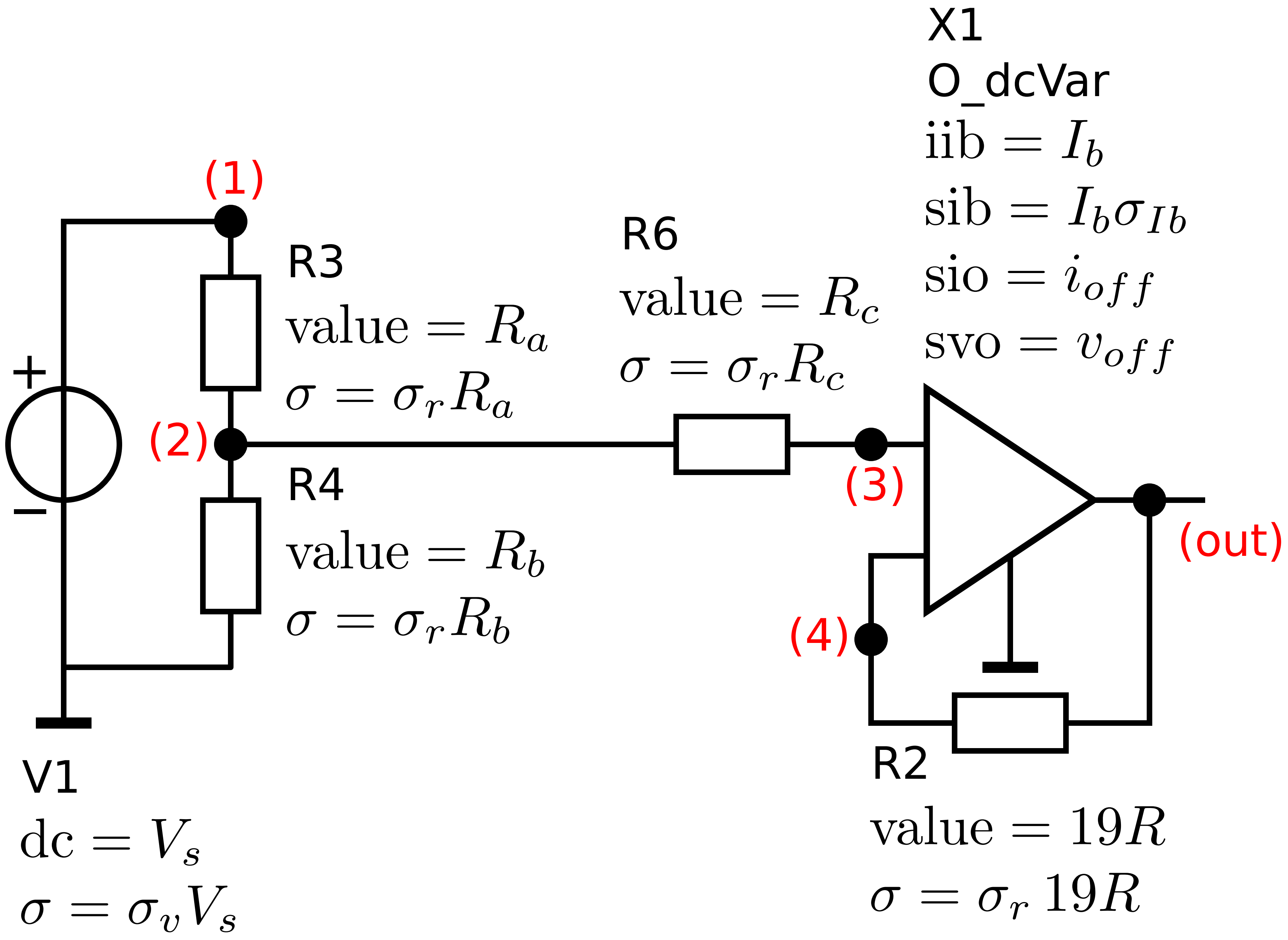


Total bias errors



Total bias errors

Simplified result: $R_c \gg \frac{R_a R_b}{R_a + R_b}$



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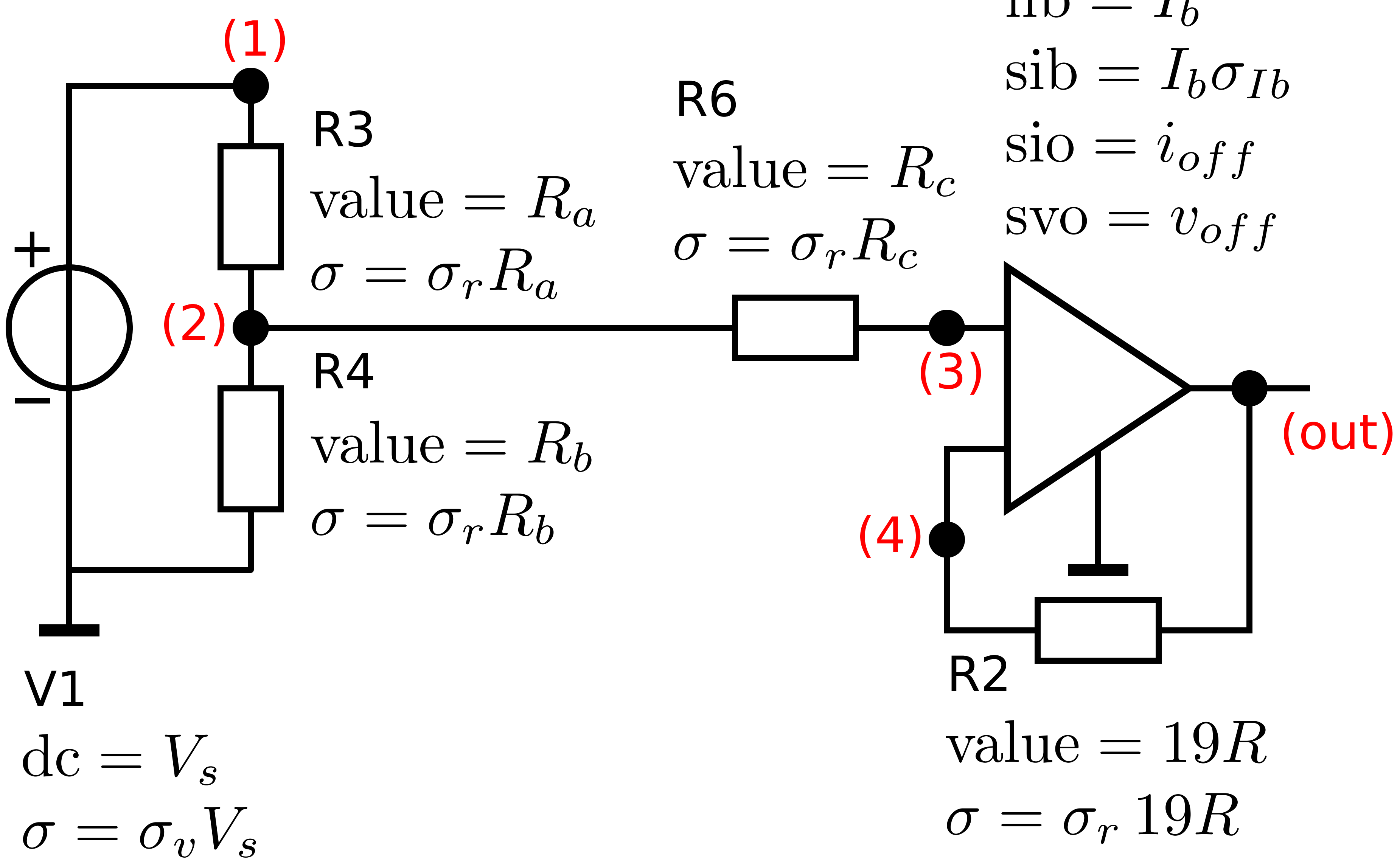

Simplified result: $R_c \gg \frac{R_a R_b}{R_a + R_b}$

$$\sigma^2_{V\,out} = 2\sigma_r^2 \left(\frac{V_s}{R_a + R_b}\right)^2 \left(\frac{R_a R_b}{R_a + R_b}\right)^2 + \sigma_v^2 V_s^2 \left(\frac{R_b}{R_a + R_b}\right)^2 + v_{off}^2 + i_{off}^2 \left(R_c + 19R\right)^2 + \sigma_{Ib}^2 I_b^2 \left(R_c - 19R\right)^2 + \sigma_r^2 I_b^2 \left(R_c^2 + (19R)^2\right)$$