Modeling of Negative Feedback Amplifiers

Black, Superposition Model, Asymptotic-gain Model

Anton J.M. Montagne

Perscitec B.V.

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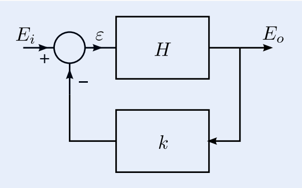
Introduction

History of feedback

- 1927 Harold Black (1898-1983) built the first negative-feedback amplifier
- Theory concerning stability: Harry Nyquist (1889-1976) and Hendrik Bode (1905-1982)
- 1937 Black's patent for negative feedback awarded.
- Nowadays: widespread use of Black's feedback model
- Not optimally suited for two-step design:
  1. Design of the ideal transfer: feedback network(s) around nullor(s)
  2. Design of the *controller(s)* also: *loop amplifier(s)*.

Introduction

Black's feedback model



E_i = input quantity
E_o = output quantity
H = gain of high-gain amplifier
k = transfer feedback network
ε = error signal
$1 + Hk$ = return difference
Hk = return ratio also: *loop gain*

Input-output transfer

$$\frac{E_o}{E_i} = \frac{1}{k}\left(\frac{Hk}{1+Hk}\right), \quad \lim_{Hk \to \infty}\left(\frac{E_o}{E_i}\right) = \frac{1}{k}$$

Introduction

Black's feedback model

Not modeled:

- Influence of source and load impedances
- Direct transfer from source to load
- Influence of impedance feedback network

Suited for design with voltage operational amplifiers:

- High differential and common-mode input impedance
- High common-mode rejection
- Low output impedance

Model does not facilitate the two-step design approach for application-specific negative-feedback amplifiers

Selection of the loop gain reference

SLiCAP examples application of Black's feedback model

- Section 6.2.2. in the book, example 6.1 and 6.2
- SLiCAP execute: `~/day5/Exercises5/Exercises5.m`
- Study script and its output: `~/day5/Exercises5/vAmpBlack.m`
- Study script and its output: `~/day5/Exercises5/vAmpBlackF.m`

Feedback modeling

Hendrik Bode, 1945

- In 1945, Hendrik Bode formulated a feedback theory based on the selection of a controlled source as loop gain reference:
  The *Return Difference F* and the *Return Ratio T* for the specific selection of a controlled source can be found from the network matrix equation as:

$$F = 1 + T = \frac{\Delta}{\Delta^0}$$

Where: Δ is the determinant of the network matrix and Δ^0 is the determinant of the network matrix in which the gain of the selected controlled source has been set to 0.

Feedback modeling

Stability

- The return ratio or loop gain is a complex transfer function: $T(s)$. If its value is larger than unity, we speak of negative, degenerated or corrected feedback.
- Sufficiently stable behavior of the *return difference* requires a certain distance of $T(s)$ to -1
  - *phase margin* and *amplitude margin* tell us about this distance, and thus about the 'amount of stability' with the selected loop gain reference
- However, they do not tell us anything about the actual frequency response of the amplifier:
  - How does the transfer of the amplifier relate to the return difference or to the loop gain for an arbitrarily selected loop gain reference?

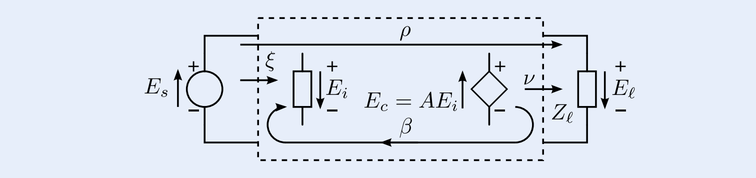
Feedback modeling

Rosenstark

- The asymptotic-gain model, described by Rosenstark provides relations between the return difference and the source to load transfer of an amplifier
  - It is based on the superposition model
  - The expression for the source to load transfer obtained with this model is identical to one obtained from the cofactors of the circuit network
- The asymptotic-gain model facilitates a two-step approach for the design of negative feedback amplifiers in the best possible way:
  1. Design of the ideal transfer (nullors as controllers)
  2. Design of the controller

Superposition Model

Introduction



The gain of one arbitrarily selected controlled source of the network is selected as reference variable:

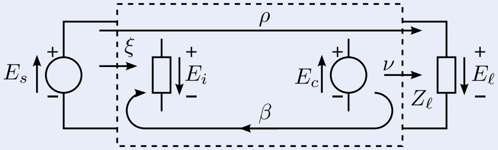
$$A = \frac{E_c}{E_i}$$

E_c = controlled quantity, E_i = controlling quantity

Superposition Model

Basic equations

The controlled source is replaced with an independent source E_c.



The dependent variables E_i and E_ℓ can now be written as a linear combination of the independent variables E_s and E_c (superposition).

$$\begin{bmatrix} E_\ell \\ E_i \end{bmatrix} = \begin{bmatrix} \rho & \nu \\ \xi & \beta \end{bmatrix} \begin{bmatrix} E_s \\ E_c \end{bmatrix}$$

Superposition Model

Model parameters and source-load transfer

Model parameters:

- $\rho = \left.\frac{E_\ell}{E_s}\right|_{E_c=0}$: Direct transfer from source to load
- $\nu = \left.\frac{E_\ell}{E_c}\right|_{E_s=0}$: Transfer from E_c to the load
- $\xi = \left.\frac{E_i}{E_s}\right|_{E_c=0}$: Transfer from source to driving quantity E_i
- $\beta = \left.\frac{E_i}{E_c}\right|_{E_s=0}$: Transfer from E_c to E_i; a non-zero value of β indicates the existence of feedback ($A\beta =$ loop gain).

source-load transfer:

$$A_f = \frac{E_\ell}{E_s} = \rho + \frac{\nu \xi A}{1 - A\beta}$$

Asymptotic-gain Model

Model derived from superposition model

Asymptotic-gain is source-load transfer when $A \to \infty$; with $\beta \neq 0$ this implies $A\beta \to -\infty$:

$$A_{f\infty} = \lim_{A\beta \to -\infty} \frac{E_\ell}{E_s} = \rho - \frac{\nu\xi}{\beta}$$

Rewrite the source-load transfer in terms of $A_{f\infty}$, ρ and $A\beta$:

$$A_f = A_{f\infty}\frac{-A\beta}{1-A\beta} + \frac{\rho}{1-A\beta} = A_{f\infty}\left(\frac{-A\beta}{1-A\beta}\right)\left(1 + \frac{\rho}{A_{f\infty}}\frac{1}{1-A\beta}\right)$$

The contribution of the direct transfer can almost always be neglected: with proper selection of A, $A_{f\infty}$ will equal the ideal gain of the amplifier. This gain will always be larger than the direct transfer!

$$A_f = A_{f\infty}\frac{-A\beta}{1-A\beta}$$

Asymptotic-gain Model

Asymptotic-gain versus ideal gain

- Asymptotic gain: $A_{f\infty} = \rho - \frac{\nu\xi}{\beta}$
- The ideal gain is the source load transfer when the controller has nullor properties
- Consider the controlling quantity of the loop gain reference is taken at the input port of the controller:

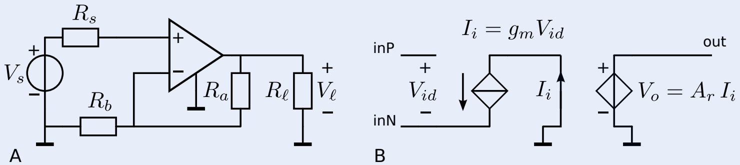
$$E_i = \frac{\xi}{1 - A\beta} E_s, \quad \lim_{A\beta \to \infty} E_i = 0$$

- If $A\beta \to \infty$ results in both zero voltage across the input port and zero current through the input port of the controller, the asymptotic gain equals the ideal gain (controller has nullor properties)
- We then have a two-step design approach for the negative feedback amplifier!
- This requires proper selection of A!

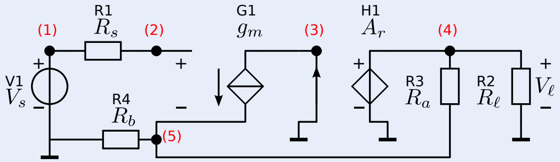
Example voltage amplifier

Negative feedback amplifier with current feedback operational amplifier

Amplifier + Opamp model:



Complete small-signal model:



Selection of the loop gain reference

SLiCAP example with current-feedback operational amplifier

- Section 6.3.3 in the book, example 6.3
- SLiCAP execute: `~/day5/Exercises5/Exercises5.m`
- Study script and its output: `~/day5/Exercises5/cfbVamp.m`

Asymptotic-gain Model

Conclusions

1. The asymptotic-gain model gives exactly the same results for the source-load transfer as MNA, this result does not depend on the choice of the reference variable
2. If the reference variable is selected in such a way that the active part of the amplifier obtains nullor properties for $A\beta \rightarrow \infty$:
   1. The asymptotic-gain equals the ideal gain
   2. If the reference variable has been selected properly, the error of the transfer with respect to this ideal gain can be described solely by the return ratio.
3. In that case the design of a negative feedback amplifier can be performed in two successive and independent steps:
   1. Design of the ideal gain (feedback topology)
   2. Design of the loop gain function $A\beta$ such that the error with respect to the ideal gain is sufficiently small. All errors with respect to the ideal gain can then be obtained from $A\beta$!

Asymptotic-gain Model

The servo function

- The servo function $F_s(s)$ is a measure for the correspondence between the gain and the asymptotic gain.
- It is defined as the ratio between the source to load transfer A_f and the asymptotic gain $A_{f\infty}$.

$$F_s = \frac{A_f}{A_{f\infty}} = \frac{-A\beta}{1 - A\beta}$$

- Its ideal value is 1.
- The relative error of the gain with respect to the asymptotic gain is the deviation of this servo function from unity:

$$\delta = F_s - 1 = \frac{-1}{1 - A\beta} \approx \frac{1}{A\beta}$$

- Pease notice: negative feedback: $A\beta < 0$. Hence $A\beta = -T$, where T is the return ratio according to the definition given by Bode.

Asymptotic-gain Model

Mismatch between ideal gain and the asymptotic-gain

- If the reference variable has been selected properly there can still be a mismatch between the ideal gain and the asymptotic-gain
- Parasitic impedances between the input port and/or the output port of the nullor and ground may compromise the two-port port conditions.
- Section 6.3.3 in the book, example 6.4
- SLiCAP execute: `~/day5/Exercises5/Exercises5.m`
- Study script and its output: `~/day5/Exercises5/cfbVampExtended.m`

Asymptotic-gain Model

Impedance model

- The asymptotic-gain model can also be used for the design of accurate, finite, non-zero port impedances
- Realization of such impedances requires both series and shunt feedback at an amplifier port
- If a port is shorted, the shunt feedback is inactive and the loop gain equals $A\beta_{sc}$
- If a port is left open, the series feedback is inactive and the loop gain equals $A\beta_o$
- The port impedance can be written in terms of its asymptotic-gain $Z_{f\infty}$ and the two loop gains $A\beta_{sc}$ and $A\beta_o$.

Asymptotic-gain Model

Impedance model

- Loop gain with port terminals shorted: $A\beta_{sc}$
- Loop gain with port terminals left open: $A\beta_{o}$
- Port impedance of amplifier concept with nullor: $Z_{f\infty}$
- Actual port impedance:

$$Z_f = Z_{f\infty} \frac{A\beta_o}{1 - A\beta_o} \frac{1 - A\beta_{sc}}{A\beta_{sc}}$$

- We have two servo functions, one applies to the open port and the other to the shorted port.

Asymptotic-gain model

Influence of negative feedback on the port impedances of single-loop feedback amplifiers

- $Z_{f\infty}$ = ideal value of the port impedance; single loop negative feedback:
  - parallel feedback at the port $Z_{f\infty} = 0$
  - series feedback at the port $Z_{f\infty} = \infty$
- Usually no need for studying the port impedance Z_f of single-loop negative feedback amplifiers:
  - All error contributions including those due to non ideal port impedances are included in $A\beta$.
- If desired, the port impedance with feedback Z_f can be found from impedance model:
  - Shunt feedback at the port: $A\beta_{sc} = 0$: $Z_f = \frac{\rho}{1-A\beta_o}$
  - Series feedback at the port: $A\beta_o = 0$: $Z_f = \frac{\rho}{1-A\beta_{sc}}$
  - ρ is the port impedance with the loop gain reference set to zero.

Impedance model

Obtain output impedance operational amplifier from its 'closed-loop' value and the loop gain

- Shunt feedback at the port:

$$Z_f = \frac{\rho}{1 - A\beta_o}$$

ρ is the port impedance Z_x with the loop gain reference set to zero, we obtain:

$$\begin{aligned} Z_x &= Z_f\left(1 - A\beta_o\right) \\ |A\beta_o| \gg 1 \\ &\approx -A\beta_o Z_f \end{aligned}$$