

Negative Feedback Amplifier Configurations

Design of negative feedback amplifier configurations

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Perscitech

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Negative feedback: an error reduction technique

Design of high-performance negative-feedback amplifier configurations

Introduction

Design of
feedback
amplifiers
configurations

Passive
feedback
amplifiers

Active
feedback

Balancing and
port isolation

Indirect
feedback

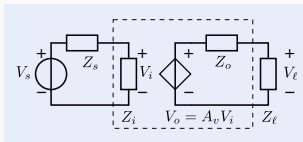
Feedback
configurations
with OpAmps

- Negative feedback trades available power gain of amplifying devices for quality improvement of the information transfer
- Port impedances and transfer characteristics are fixed by feedback network(s)
- Amplifier performance aspects can be designed independently:
 - Port impedances and transfer characteristics
 - Accuracy
 - Noise performance
 - Bandwidth
 - Linearity
 - Power efficiency

Port impedances

Influence of source and load impedance on transfer

Consider voltage amplifier driving a load Z_ℓ , and driven from V_s with impedance Z_s .



- Z_s and Z_ℓ are part of transfer if $Z_i \neq \infty$ and $Z_o \neq 0$:

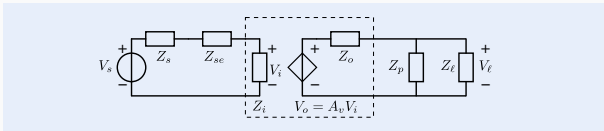
$$\frac{V_\ell}{V_s} = \frac{Z_i}{Z_i + Z_s} A_v \frac{Z_\ell}{Z_\ell + Z_o}$$

- Their inaccuracy, nonlinearity and dynamic behavior may deteriorate the quality of the source to load transfer

Brute force impedance realization

Deterioration of the signal-to-noise ratio

- Influences of Z_s and Z_ℓ can be reduced by placing linear and accurate impedance in series and/or in parallel with the signal path

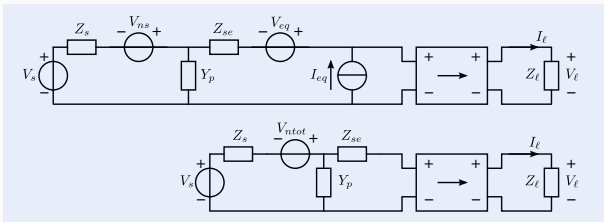


- These techniques are called *brute force techniques*
- Their application may result in deterioration of the signal-to-noise ratio and the power efficiency of the amplifier.

Brute force impedance realization

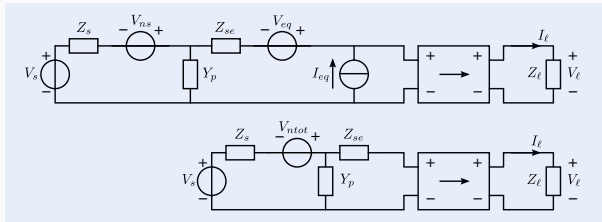
Deterioration of the signal-to-noise ratio

- Consider amplifier with equivalent noise sources V_{eq} and I_{eq}
- The amplifier is driven from a voltage source V_s with source impedance Z_s .
- The noise associated with the source impedance is V_{ns} .
- An impedance Z_{se} has been placed in series with the signal path
- An admittance Y_p has been placed in parallel with the signal path:



Brute force impedance realization

Deterioration of the signal-to-noise ratio



$$S_{V_{ntot}} = 4kT \left[\{ \text{Re}(Z_s) \} + \{ \text{Re}(Z_{se}) \} |1 + Z_{se} Y_p|^2 + \{ \text{Re}(Y_p) \} |Z_s|^2 \right] \\ + S_{V_{eq}} |1 + Z_{se} Y_p|^2 + S_{I_{eq}} |Z_{se} + Z_s (1 + Z_{se} Y_p)|^2.$$

Only in narrow-band applications the noise can be improved by inserting impedances in series or in parallel with the signal path.

Brute force impedance realization

Deterioration of the signal-to-noise ratio

At the input port, impedances in series and/or in parallel with the source will deteriorate the signal to noise ratio.

This deterioration also has two causes:

- Impedances in series and/or in parallel with the source increase the contribution of the equivalent input noise sources of the amplifier
 - Only in narrow-band applications, improvement can be achieved by tuning-out unwanted effects from reactive parts of impedances in series or in parallel with the source
- If the impedances in series and/or in parallel with the signal source have a real part they add noise.

Brute force impedance realization

Deterioration of the power efficiency

At the output, impedances in series and/or in parallel with the load will deteriorate the power efficiency of the amplifier.

This deterioration also has two causes:

- Impedances in series and/or in parallel with the load require the amplifier's output current or voltage excursions to be larger for the same amount of load power.
 - This results in an increase of the power dissipation for any amplifier with less than 100% power efficiency
 - Only in narrow-band applications, improvement can be achieved by tuning-out unwanted effects from reactive parts of impedances in series and/or in parallel with the load
- If the impedances in series and/or in parallel with the load have a real part, extra power has to be delivered by the amplifier.

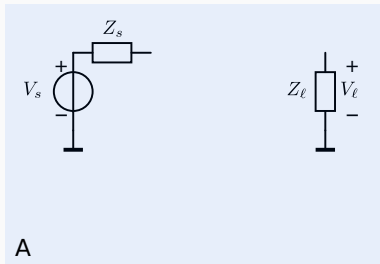
The principle of negative feedback

Sensing of load quantity and comparison with source quantities

- ① Sensing of the load quantity that needs to be related to a source quantity
- ② Converting the sense result into a copy of the source quantity to which it should be related:
 - Multiplication with the desired value of the corresponding transmission-1 two-port matrix coefficient
- ③ Nullifying the error between the actual source quantity and the derived copy, with the aid of a high-gain controller
 - Feedback elements primarily define the gain, the accuracy, the linearity and the bandwidth of a negative feedback amplifier
 - The performance aspects of the controller only become important if error signal cannot be maintained zero
 - Design of negative-feedback amplifiers is performed in two steps:
 - ① Design of the feedback network(s): The amplifier feedback configuration
 - ② Design of the controller (also: error amplifier).

Design of the amplifier type

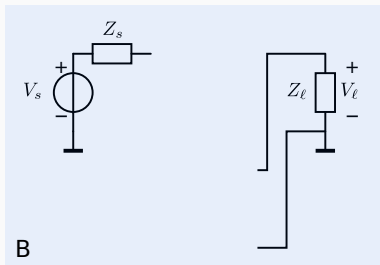
We want to relate a load voltage to a source voltage



At the start of the process we know the desired value of the load quality in response of the source quantity.

Design of the amplifier type

Sensing of the load voltage

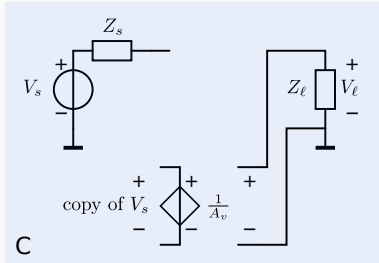


We will sense the value of the load quantity, derive a copy of the source quantity from it, compare this copy with the source quantity and nullify the difference between this copy and the source quantity.

Here we apply the first step: sensing of the load quantity. Sensing of a voltage requires a parallel connection to this voltage.

Design of the amplifier type

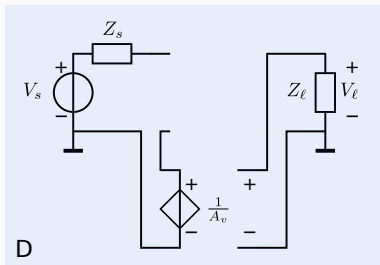
Generation of a copy of the source voltage



We obtain a copy of the source quantity by multiplying the sensed load quantity with the transmission 1 matrix parameter that needs to be fixed by feedback. In this case we multiply with $A = \frac{1}{A_v}$

Design of the amplifier type

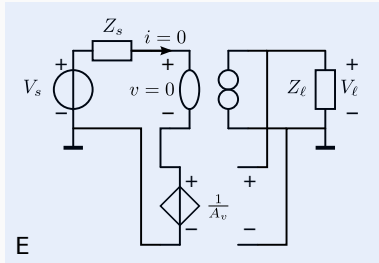
Comparison of the obtained copy with the actual source voltage



Comparison of two voltages is done by subtraction. This requires anti-series connection of the two voltages.

Design of the amplifier type

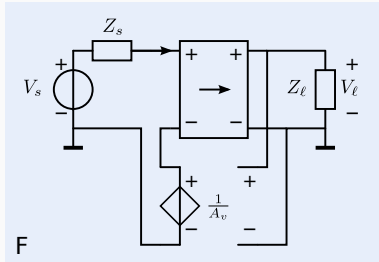
Nullify the difference use a nullor



The result of the comparison can be nullified by placing a nullator between them. A network solution is obtained if we also insert a norator. This should be placed in such a way that it does not alter the condition set by the nullor and that this condition can be achieved through inserting of a current by the nullator.

Design of the amplifier type

Replace nullor with high-gain amplifier, arrange negative feedback

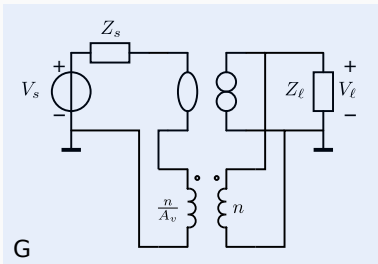


At a later stage of the design, the nullor needs to be replaced with a real-world controller. A practical controller will suffer from noise, speed and power limitations. Negative feedback, also *degenerative* or *corrective* feedback is established if the transfer in the loop has a negative sign.

Design of the amplifier type

feedback

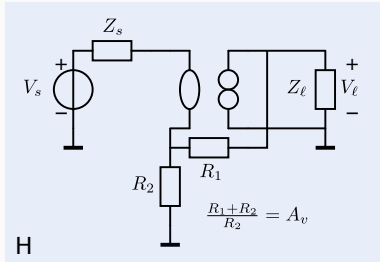
Nonenergetic



The feedback element determines the type, the value and the sign of the transfer. Here, we used a transformer, a nonenergetic feedback element.

Design of the amplifier type

Passive feedback



We speak of passive feedback if the feedback network comprises passive elements only. For this voltage amplifier we used a passive voltage divider (also: *passive voltage attenuator*) as feedback network.

Types of negative feedback

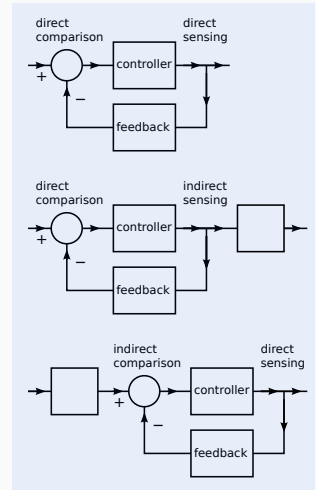
We distinguish two types of feedback:

① Direct feedback:

- Sensing of desired output port quantity
- Comparison with source quantity

② Indirect or model-based feedback

- Indirect sensing: sensing of model or copy of output port quantity
- Indirect comparison: comparison with copy or model of input port quantity



Implementation of feedback

- Nonenergetic feedback: feedback network(s) comprise nonenergetic network element(s) only
- Passive feedback: feedback networks with passive networks only
- Active feedback: feedback network(s) comprise (feedback) amplifiers
- Balanced feedback: at least one of the ports is not referred to ground

Design of feedback configurations

Feedback network design rules for negative amplifiers

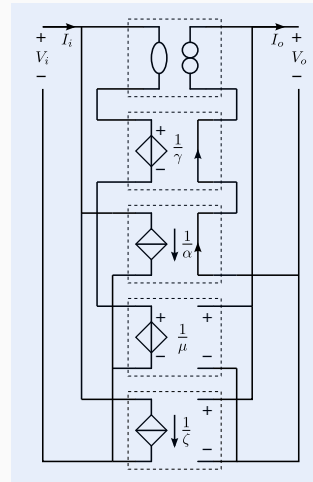
Each feedback loop fixes one of the transmission parameters:

Parameter to be fixed	Feedback type at source	Feedback type at load
<i>A</i>	V - comparison (series feedback)	V - sensing (parallel feedback)
<i>B</i>	V - comparison (series feedback)	I - sensing (series feedback)
<i>C</i>	I - comparison (parallel feedback)	V - sensing (parallel feedback)
<i>D</i>	I - comparison (parallel feedback)	I - sensing (series feedback)

Negative feedback amplifiers

Ideal gain

- The *ideal gain* of a negative feedback amplifier is the source load transfer when active part(s) are nullor(s)
- Figure: Amplifier with A , B , C and D independently fixed by negative feedback
 - No current drawn by voltage sense elements
 - No voltage drop across current sense elements
 - Zero output impedance of voltage comparison circuit
 - Infinite output impedance of current comparison circuit
 - Interaction between feedback loops if this is not the case.



Amplifier type	A	B	C	D
Nullor	0	0	0	0
Voltage amplifier (unilateral)	\times	0	0	0
Transadmittance amplifier (unilateral)	0	\times	0	0
Transimpedance amplifier (unilateral)	0	0	\times	0
Current amplifier (unilateral)	0	0	0	\times
$Z_i = \alpha/\gamma$, $Z_o = 0$ (unilateral)	0	\times	0	\times
$Z_i = \zeta/\mu$, $Z_o = \infty$ (unilateral)	\times	0	\times	0
$Z_i = 0$, $Z_o = \zeta/\alpha$ (unilateral)	0	0	\times	\times
$Z_i = \infty$, $Z_o = \mu/\gamma$ (unilateral)	\times	\times	0	0
Transformer-like amplifier (non-unilateral)	\times	0	0	\times
Gyrator-like amplifier (non-unilateral)	0	\times	\times	0
Triple loop 1 (non-unilateral)	\times	\times	\times	0
Triple loop 2 (non-unilateral)	\times	\times	0	\times
Triple loop 3 (non-unilateral)	\times	0	\times	\times
Triple loop 4 (non-unilateral)	0	\times	\times	\times
Quadruple loop	\times	\times	\times	\times

No deterioration of S/N ratio and power efficiency by feedback networks

Nonenergetic

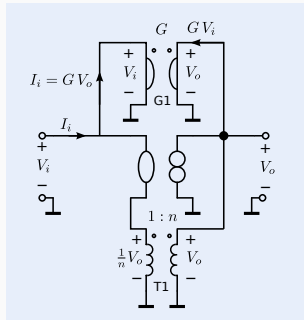
Nonenergetic **network elements**

- No power dissipation: lossless
- No energy storage: instantaneous
 - ① Short circuit (one port, $V = 0$)
 - ② Open circuit (one port, $I = 0$)
 - ③ Ideal transformer (two port, $A = \frac{1}{n}$, $B = 0$, $C = 0$, $D = n$)
 - ④ Ideal gyrator (two-port, $A = 0$, $B = \frac{1}{G}$, $C = G$, $D = 0$)

feedback

Dual loop nonenergic negative-feedback amplifier

Nonenergic



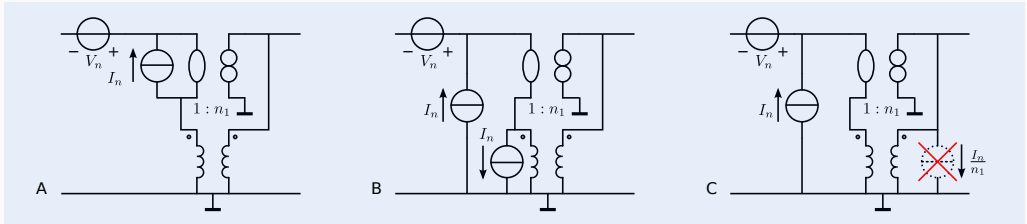
- Transmission-1 parameter A fixed by the turns ratio of the transformer
- Transmission-1 parameter C fixed by the gyrator transfer
- No interaction between the loops

feedback

S/N ratio and Power Efficiency of nonenergic negative feedback amplifiers

Nonenergic

The signal-to-noise ratio and the power efficiency of nonenergic feedback amplifiers is equal or better (in case of non unilateral transfer) to that of its active part.

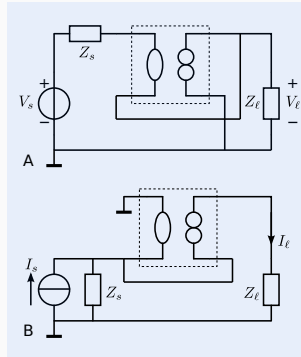


feedback

Voltage and current followers (3-terminal)

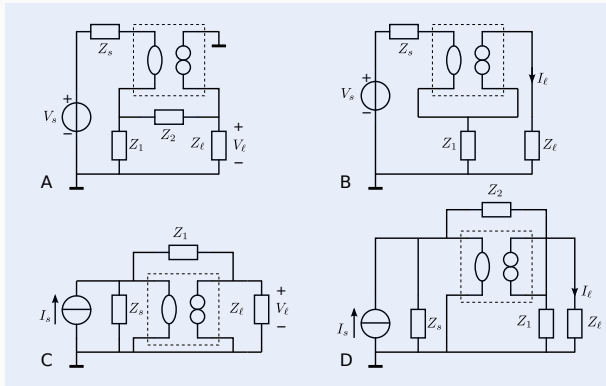
Nonenergetic

- Practical application of nonenergetic feedback: voltage and current follower
- Feedback networks consist of short and open circuit



Passive feedback

Single-loop, direct, passive feedback configurations



- ① Voltage amplifier:
 $\frac{V_\ell}{V_s} = \frac{1}{A} = \frac{Z_1 + Z_2}{Z_1}$
- ② Transadmittance amplifier:
 $\frac{I_\ell}{V_s} = \frac{1}{B} = -\frac{1}{Z_1}$
- ③ Transimpedance amplifier:
 $\frac{V_\ell}{I_s} = \frac{1}{C} = -Z_1$
- ④ Current amplifier:
 $\frac{I_\ell}{I_s} = \frac{1}{D} = \frac{Z_1 + Z_2}{Z_1}$

Single-loop direct passive feedback

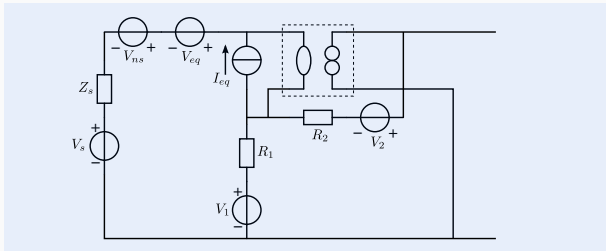
Conclusions with respect to possible configurations

- Voltage and the current amplifier have non-inverting transfers
- Transadmittance and transimpedance amplifiers have inverting transfers
- No port isolation (3-terminal networks)
- "Lost configurations" can be regained through application of:
 - Active feedback
 - Balancing
 - Indirect feedback
- This will be discussed later

Single-loop direct passive feedback

Noise behavior

Noise model of single-loop passive-feedback voltage amplifier



V_{ns} : Noise associated with signal source

V_{eq} : Equivalent input voltage noise of controller

I_{eq} : Equivalent input current noise of controller

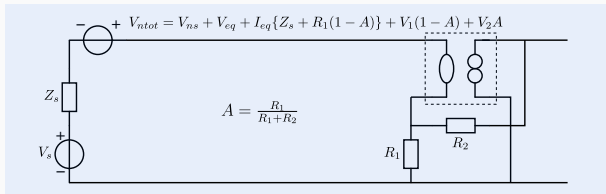
V_1 : Voltage noise of R_1

V_2 : Voltage noise of R_2

Single-loop direct passive feedback

Noise behavior

Equivalent noise model of single-loop passive-feedback voltage amplifier

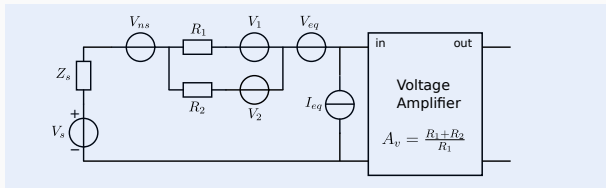


$$S_{V_{n,tot}} = 4kT \operatorname{Re}\{Z_s\} + S_{veq} + 4kT \frac{R_1 R_2}{R_1 + R_2} + S_{ieq} \left| Z_s + \frac{R_1 R_2}{R_1 + R_2} \right|^2$$

Single-loop direct passive feedback

Noise behavior

Simplified equivalent noise model of single-loop passive-feedback voltage amplifier



- Noise performance degradation as if the parallel connection of the feedback resistors is in series with the signal source

Single-loop direct passive feedback

Noise behavior, conclusions

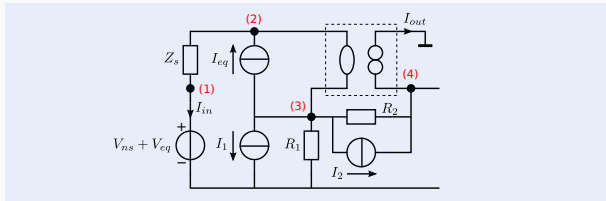
Conclusions with respect to the noise behavior of the other single-loop passive feedback amplifier configurations are obtained in a similar way:

- ① The total equivalent input noise of the voltage amplifier is affected by the feedback elements as if their parallel connection is in series with the source.
- ② The total equivalent input noise of the transadmittance amplifier is affected by the feedback element as if it is in series with the source.
- ③ The total equivalent input noise of the transimpedance amplifier is affected by the feedback element as if it is in parallel with the source.
- ④ The total equivalent input noise of the current amplifier can be calculated as if their series connection is in parallel with the source.

Single-loop direct passive feedback

Noise behavior, use of MNA

Alternative way of finding the total equivalent input noise:



MNA equations:

$$\begin{pmatrix} 0 & I_{eq} & -I_1 - I_2 - I_{eq} & I_2 & V_{eq} + V_{ns} & 0 \end{pmatrix}^T \\ = \mathbf{M} \begin{pmatrix} V_1 & V_2 & V_3 & V_4 & I_{in} & I_{out} \end{pmatrix}^T$$

Single-loop direct passive feedback

Noise behavior, use of MNA

$$\mathbf{M} = \begin{pmatrix} \frac{1}{Z_s} & -\frac{1}{Z_s} & 0 & 0 & 1 & 0 \\ -\frac{1}{Z_s} & \frac{1}{Z_s} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

The total equivalent output noise voltage V_{no} can be found from:

$$S_{V_{no}} = \left| \frac{\det \mathbf{N}}{\det \mathbf{M}} \right|_{s=j\omega}^2$$

Single-loop direct passive feedback

Noise behavior, use of MNA

The output voltage equals V_4 ; it can be found using Cramer's rule:

$$\mathbf{N} = \begin{pmatrix} \frac{1}{Z_s} & -\frac{1}{Z_s} & 0 & 0 & 1 & 0 \\ -\frac{1}{Z_s} & \frac{1}{Z_s} & 0 & I_{eq} & 0 & 0 \\ 0 & 0 & \frac{1}{R_1} + \frac{1}{R_2} & -I_1 - I_2 - I_{eq} & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2} & I_2 & 0 & 1 \\ 1 & 0 & 0 & V_{eq} + V_{ns} & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix}$$

The total equivalent input voltage is obtained as:

$$S_{V_{ni}} = S_{V_{no}} \left| \frac{R_1}{R_1 + R_2} \right|^2$$

$$S_{V_{ni}} = S_{V_{eq}} + S_{V_{ns}} + S_{I_{eq}} \left| \frac{R_1 R_2}{R_1 + R_2} + Z_s \right|^2 + (S_{I_1} + S_{I_2}) \left| \frac{R_1 R_2}{R_1 + R_2} \right|^2$$

SLiCAP noise analysis

- See: SLiCAP/vAmpNoise/html/index.html

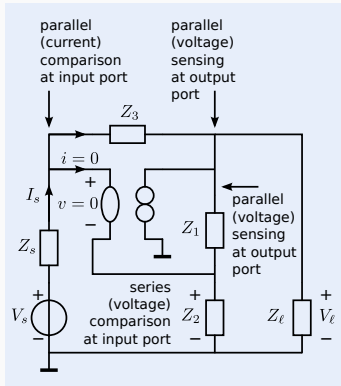
Single-loop direct passive feedback

Power efficiency of single-loop passive feedback configurations

- ① The power efficiency voltage amplifier is affected by the feedback elements as if their series connection is in parallel with the load impedance
- ② The power efficiency of the transadmittance amplifier is affected by the feedback element as if it is in series with the load impedance
- ③ The power efficiency of the transimpedance amplifier is affected by the feedback element as if it is in parallel with the load impedance
- ④ The power efficiency of the current amplifier is affected by the feedback elements as if their parallel connection is in series with the load impedance.

Nonzero finite input impedance, zero output impedance

- Fix A and C using negative feedback



$$A = \frac{Z_2}{Z_1 + Z_2}$$

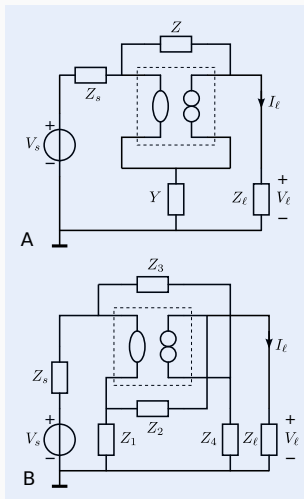
$$C = -\frac{Z_1}{Z_1 + Z_2} \frac{1}{Z_3}$$

$$Z_i = \frac{A}{C} = -Z_3 \frac{Z_2}{Z_1}$$

- Negative input impedance:
 - one positive feedback loop

Dual-loop direct passive feedback

Gyrator-like passive feedback configurations



Gyrator-like configuration (A):

- $A = \frac{1}{1-YZ}$, $B = \frac{Z}{1-YZ}$, $C = \frac{Y}{1-YZ}$, $D = \frac{1}{1-YZ}$
- Impedance matching at both ports if $Z_k = \sqrt{\frac{Z}{Y}}$.

Transformer-like configuration (B):

- $A = \frac{Z_1}{Z_1+Z_2}$, $B = 0$,
 $C = \frac{Z_1+Z_4}{(Z_1+Z_2)(Z_3+Z_4)}$, $D = \frac{Z_4}{Z_3+Z_4}$

Active feedback

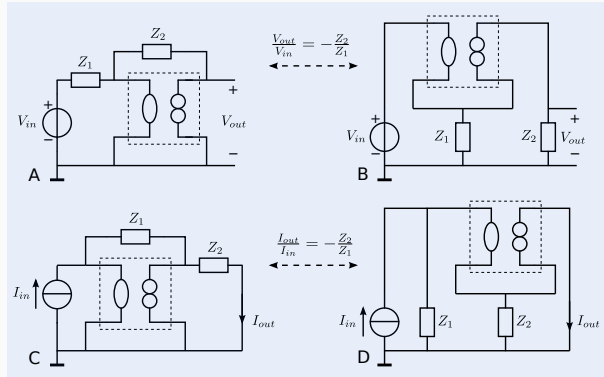
Introduction

- Unbalanced passive feedback: no signal inversion in passive attenuators
 - Sign of transfer is result of passive feedback connection
- Signal inversion in feedback network required for realization of unbalanced:
 - Inverting voltage amplifiers
 - Inverting current amplifiers
 - Non inverting transimpedance
 - Non inverting transadmittance
- Use active circuits based on passive-feedback inverting transadmittance or transimpedance amplifiers.

Active feedback

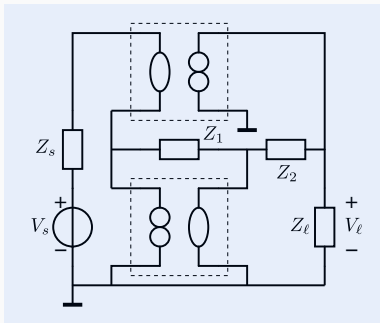
Inverting voltage and current attenuators

- Figure shows inverting voltage and current attenuators based on the inverting transimpedance and transadmittance
- Inverting transimpedance and transadmittance already known



Single-loop active feedback

Inverting voltage amplifier

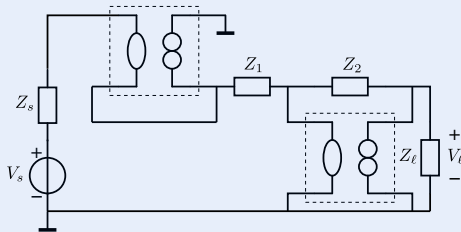


- Noise behavior and power efficiency closely matches corresponding passive feedback non inverting configuration
- Other configurations can be designed similarly

Single-loop active feedback

Inverting voltage amplifier, alternative solution

- Different pairing of nullators and norators yields an alternative solution



Dual-loop active feedback

Introduction

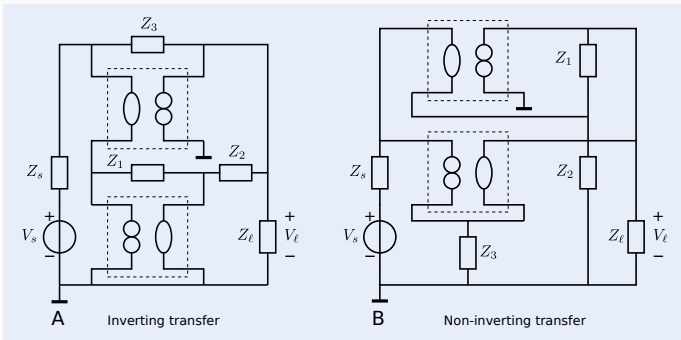
- Unilateral configurations with either Z_i or Z_{out} accurately fixed:

Amplifier type	A	B	C	D
$Z_i = B/D, Z_o = 0$ (unilateral)	0	B	0	D
$Z_i = A/C, Z_o = \infty$ (unilateral)	A	0	C	0
$Z_i = 0, Z_o = D/C$ (unilateral)	0	0	C	D
$Z_i = \infty, Z_o = B/A$ (unilateral)	A	B	0	0

- Unbalanced passive feedback: sign of signal transfer cannot be designed independently from type of transfer
 A and D non-inverting, B and C inverting
- Configurations require extra signal inversion in one feedback loop
- Inversion realized using transimpedance and transadmittance
- Four possible active feedback configurations for each amplifier type

Dual-loop active feedback

Examples

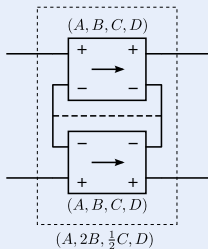


Obtain alternative configurations from pairing nullators and norators differently.

Balanced feedback amplifiers

Anti-series connection for improved port isolation

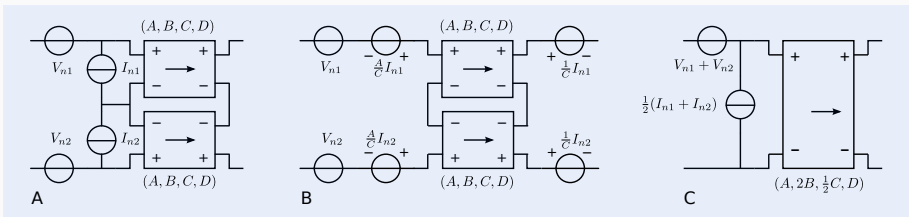
Balancing



- Create four-terminal (floating port) amplifiers using anti-series connection of three terminal amplifier
- The transmission parameters of a two-port that consists of a series (or anti-series) connection of two identical two-ports with transmission parameters A , B , C , and D , equal A , $2B$, $\frac{1}{2}C$, and D , respectively.

Noise of balanced amplifiers

Noise modeling for anti-series connected two ports

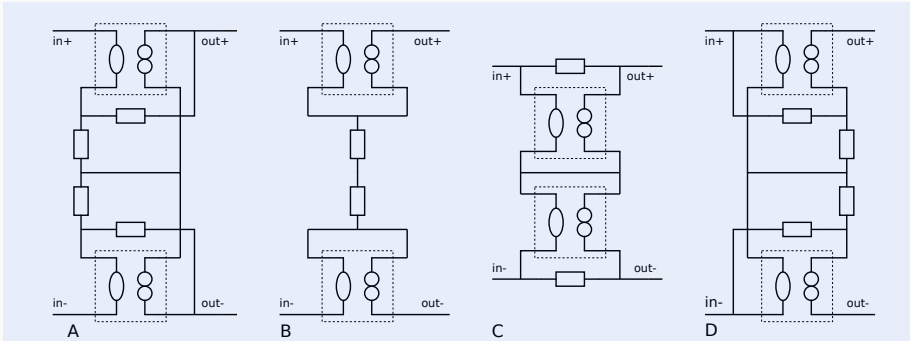


If all noise sources are uncorrelated and if $S_{V_{n1}} = S_{V_{n2}} = S_V$ and if $S_{I_{n1}} = S_{I_{n2}} = S_I$:

- The spectral density of the total equivalent noise voltage of the anti-series connection is $2S_V$
- The spectral density of the total equivalent noise current of the anti-series connection is $\frac{1}{2}S_I$

Balanced feedback amplifiers

Single-loop balanced, passive feedback amplifiers



A. Balanced voltage amplifier

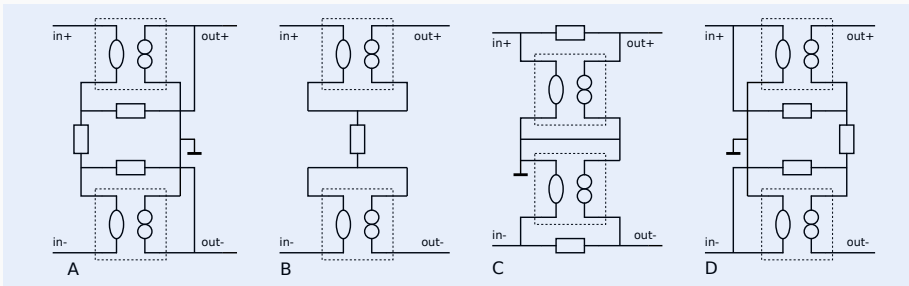
B. Balanced transadmittance amplifier

C. Balanced transimpedance amplifier

D. Balanced current amplifier

Balanced feedback amplifiers

Single-loop balanced feedback commonly used



A. Balanced voltage amplifier

B. Balanced transadmittance amplifier

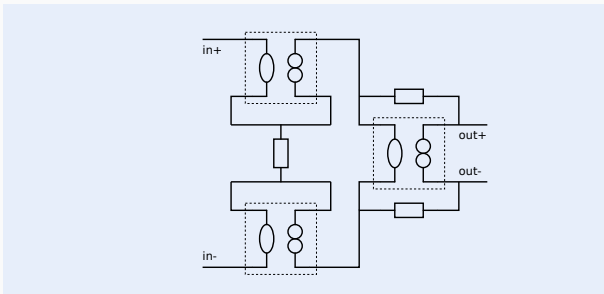
C. Balanced transimpedance amplifier

D. Balanced current amplifier

- Floating node can be connected to ground if source and load are isolated from ground.
- Otherwise possible reduction of CMRR

Balanced feedback amplifiers

Example: balanced voltage amplifiers

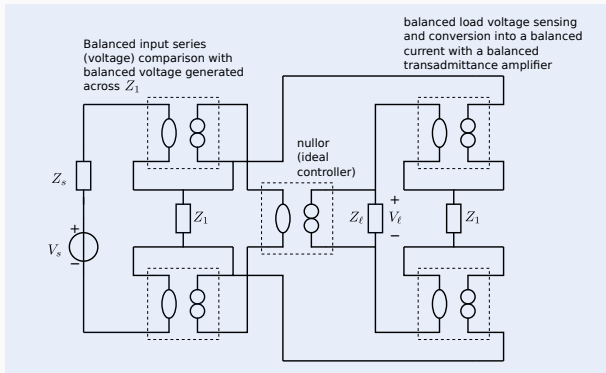


A. Balanced voltage amplifier that has no common mode transfer

B. Balanced voltage amplifier with unity gain common mode transfer

Balanced feedback amplifiers

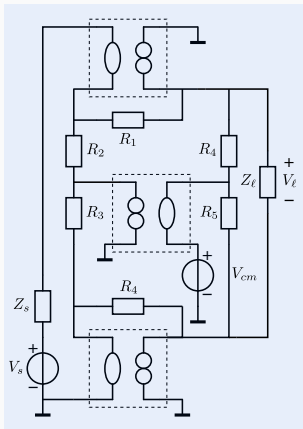
Example: high-CMRR active feedback balanced voltage amplifier



Similar topology used in Analog Devices' high CMRR AMP01

Balanced feedback amplifiers

Single-ended to differential converter with programmable common-mode output voltage



$$A_v = \frac{V_\ell}{V_s} > 2$$

$$R_2 = \frac{R_1}{A_v - 1}$$

$$R_3 = \frac{A_v R_1}{A_v^2 - 3A_v + 2}$$

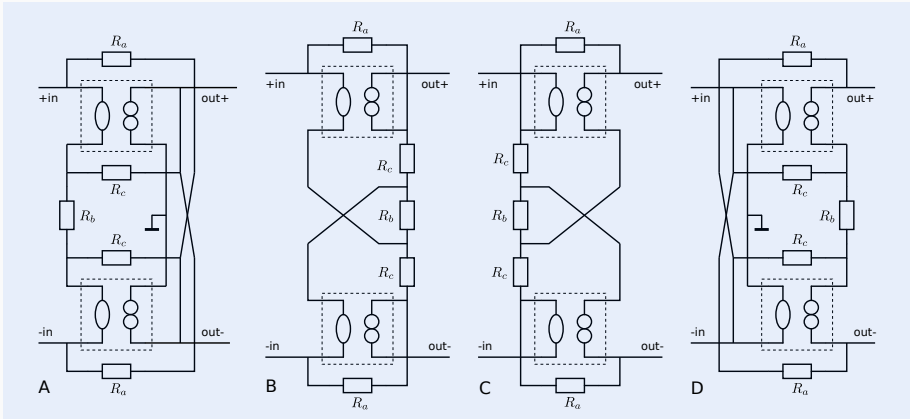
$$R_4 = \frac{A_v R_1}{A_v - 2}$$

$$R_5 = R_6$$

R_1 and R_5 selected on grounds of noise performance and power efficiency

Balanced feedback amplifiers

Dual loop balanced feedback



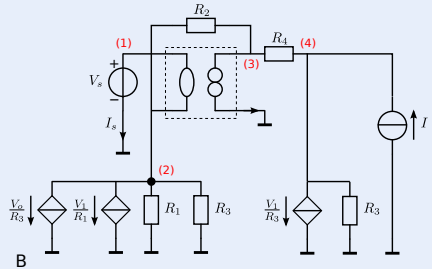
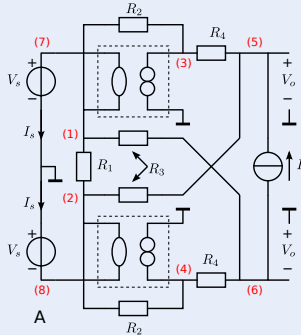
Feedback signal inversion: cross-coupled connection from output to input port

(A,B) A and C fixed, $B = D = 0$

(C,D) B and D fixed, $A = C = 0$

Balanced feedback amplifiers

Example ADSL driver with OpAmps, positive and negative output parallel feedback

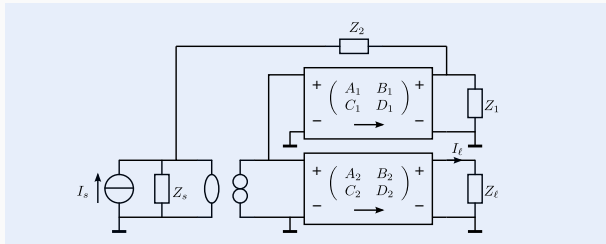


Circuit analysis with single-ended equivalent model (B)

$$A_v = \frac{R_1 (R_2 + R_3 - R_4) + 2R_2 R_3}{R_1 (R_3 + R_4 - R_2)}, \quad R_o = \frac{2R_3 R_4}{R_3 + R_4 - R_2}$$

Indirect or model-based feedback amplifiers

Example of indirect sensing

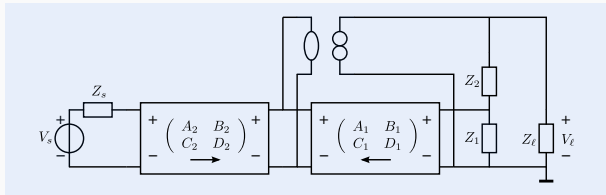


$$\frac{I_\ell}{I_s} = \frac{Z_1 + Z_2}{Z_1} \left(\frac{A \frac{Z_1 Z_2}{Z_1 + Z_2} + B}{A Z_\ell + B} \right)$$

Indirect current sensing: transfer depends on Z_ℓ

Indirect feedback amplifiers

Example of indirect comparison



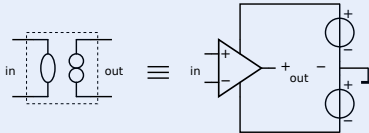
$$\frac{V_\ell}{V_s} = \frac{Z_1 + Z_2}{Z_1} \left(\frac{B + D \frac{Z_1 Z_2}{Z_1 + Z_2}}{B + D Z_s} \right)$$

Indirect voltage comparison: transfer depends on Z_s

Negative-feedback amplifiers

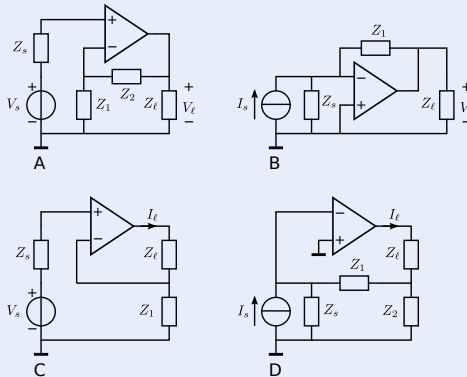
OpAmps as a controller

Limited number of configurations: one output terminal connected to power supply ground.



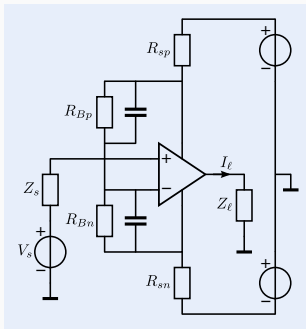
Negative-feedback amplifiers

Basic single-loop passive feedback configurations with operational amplifiers



Negative-feedback amplifiers

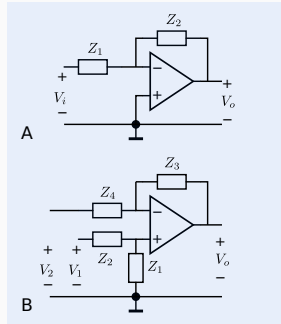
Transadmittance with grounded load



- Power supply terminals used as signal output
- Supply current sensing with transistors in follow-up course
- Mismatch in source and sink current sensing (R_{sp} , R_{sn}):
 - Offset
 - Even harmonic distortion

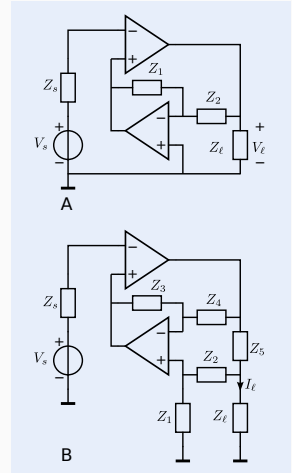
Negative-feedback amplifiers

Active feedback elements and active feedback amplifiers



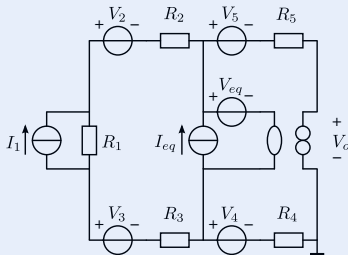
A: Inverting voltage attenuator

B: Differential to single-ended voltage conversion
(noisy)



Negative-feedback amplifiers

Noise of current sense amplifier with differential voltage amplifier



$$R_2 = R_3 = R, R_4 = R_5 = AR, R_1 \ll R$$

$$S_{V_o} = 8kTRA(1 + A) + S_{V_{eq}}(1 + A)^2 + S_{I_{eq}}(2AR)^2$$

$$S_{I_i} = 8kT \frac{R}{R_1^2} \frac{A(1 + A)}{A^2} + S_{V_{eq}} \frac{1}{R_1^2} \left(\frac{1 + A}{A} \right)^2 + S_{I_{eq}} \left(2 \frac{R}{R_1} \right)^2$$