Amplifier performance and controller requirements

Application of the asymptotic-gain model for determination of controller requirements

Anton J.M. Montagne

Persitec B.V.

April 30, 2017

Performance of negative-feedback amplifiers

Error reduction capabilities

- Negative feedback trades power gain for quality improvement.
- Transfer predominantly determined by feedback network(s)
- No reduction of errors due to:
  - Noise added by noise sources at the input of the controller, including the feedback network(s)
  - Power dissipation and energy storage in components at the output of the controller, including the feedback network(s)
  - Bandwidth limitation in transfer of feedback network(s)
  - Inaccuracy in transfer of feedback network(s)
  - Nonlinearity in transfer of feedback network(s)
  - Controller parasitic impedances not included in the idealized loop

Performance of negative-feedback amplifiers

Design of amplifier configuration

- Error contributions that cannot be reduced by negative feedback should be made acceptably small during the design of the ideal gain
- They require budgeting of:
  - Feedback network(s)
  - Controller equivalent input noise sources
  - Controller power losses
  - Controller energy storage
  - Controller parasitic port impedances

Performance of negative-feedback amplifiers

Error reduction by negative feedback, design of controller

- Inaccuracy of the amplifier transfer due to a finite loop gain can be reduced by increasing the loop gain
- Nonlinearity of the amplifier transfer due to a finite loop gain can be reduced by decreasing the differential error to gain ratio of the loop gain
- Low-pass bandwidth limitation due to a finite loop gain can be reduced by increasing the loop gain-poles product

Performance of negative-feedback amplifiers

Inaccuracy

- Transfer according to the asymptotic-gain model, ignoring the direct transfer:

$$A_f = A_{f\infty}\frac{-A\beta_{DC}}{1-A\beta_{DC}}$$

- If the asymptotic-gain equals the ideal gain, then the static inaccuracy δ_{DC} is determined by the zero frequency value of the loop gain $A\beta_{DC}$:

$$\delta_{DC} = \frac{-A\beta_{DC}}{1-A\beta_{DC}} - 1 \approx \frac{1}{A\beta_{DC}}$$

Conclusion:

In order to obtain a sufficiently low static inaccuracy with respect to the asymptotic gain, the controller must provide a sufficiently large DC gain.

Performance of negative-feedback amplifiers

Differential gain

- The differential gain $\varepsilon_{A_f}(\tilde{y})$ of the source to load transfer A_f at a load signal excursion $\tilde{y}$, is the ratio of the differential gain error of the loop gain at that excursion $\varepsilon_{A\beta}(\tilde{y})$, and the loop gain in the operating point $A\beta_Q$:

$$\varepsilon_{A_f}(\tilde{y}) \cong \frac{\varepsilon_{A\beta}(\tilde{y})}{A\beta_Q}$$

Conclusion:

In order to obtain a sufficiently low nonlinearity with respect to the asymptotic gain, the controller must provide a sufficiently low differential error to gain ratio.

Performance of negative-feedback amplifiers

Bandwidth

- Transfer according to the asymptotic-gain model, ignoring the direct transfer:

$$A_f(s) = A_{f\infty}\frac{-A\beta(s)}{1-A\beta(s)} = A_{f\infty}S(s)$$

Bandwidth definition

- The bandwidth of a negative feedback amplifier will be defined as difference between the high-frequency and the low-frequency -3 [dB] frequencies of the servo function $S(j\omega)$.
- This decouples the bandwidth definition from the desired frequency characteristics of the amplifier, fixed by the poles and the zeros of the ideal gain.

Performance of negative-feedback amplifiers

MFM bandwidth with all-pole loop gain

Assume an all-pole loop gain function $L(s)$:

$$L(s) = \frac{L_{DC}}{\prod^{n}\left(1 - \frac{s}{p_i}\right)}$$

The servo function can be written as the product of a DC term and a unity gain dynamic transfer:

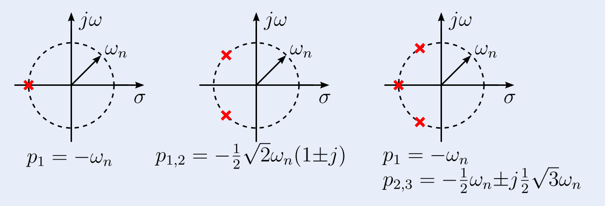
$$S(s) = \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + ... + (-1)^n \frac{s^n}{(1 - L_{DC}) \prod_{i=1}^{n} p_i}}$$

The -3dB cut-off frequency of an $n - th$ order MFM (maximally Flat Magnitude) or Butterworth low-pass filter:

$$\omega_n = \sqrt[n]{\prod_{i=1}^{n} |p_i|}$$

Performance of negative-feedback amplifiers

MFM pole positions and polynomials n=1..3



$$H_1(s) = \frac{1}{1 + s/\omega_n}$$

$$H_2(s) = \frac{1}{1 + s\sqrt{2}/\omega_n + s^2/\omega_n^2}$$

$$H_3(s) = \frac{1}{1 + 2s/\omega_n + 2s^2/\omega_n^2 + s^3/\omega_n^3}.$$

Performance of negative-feedback amplifiers

Low-pass cut off frequency of a negative-feedback amplifier

- Consider that all n poles of the loop gain $L(s)$ are dominant and can be manoeuvred into Butterworth positions
- The bandwidth of the negative feedback amplifier then equals

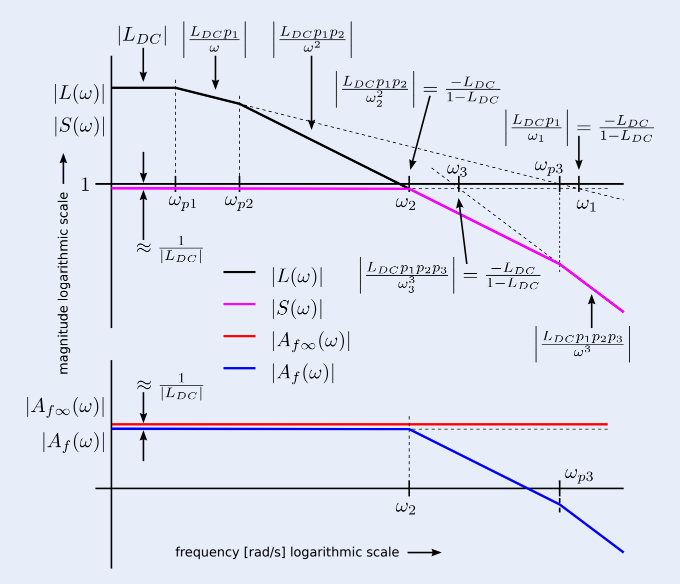
$$\omega_n = \sqrt[n]{|1 - L_{DC}| \prod_{i=1}^{n} |p_i|}$$

- From now on we will assume: $-L_{DC} \gg 1$:

$$\omega_n \approx \sqrt[n]{|L_{DC}| \prod_{i=1}^{n} |p_i|}$$

Conclusion:

In order to obtain a sufficiently large low-pass cut-off frequency with respect to the asymptotic gain, the controller must provide a sufficiently large loop gain poles product.



$|L_{DC}|$
$\left|\frac{L_{DC}p_1}{\omega}\right|$
$\left|\frac{L_{DC}p_1p_2}{\omega^2}\right|$
$\left|\frac{L_{DC}p_1p_2}{\omega_2^2}\right| = \frac{-L_{DC}}{1-L_{DC}}$
$\left|\frac{L_{DC}p_1}{\omega_1}\right| = \frac{-L_{DC}}{1-L_{DC}}$
$|L(\omega)|$
$|S(\omega)|$
1
ω_{p1}
ω_{p2}
ω_2
ω_3
ω_{p3}
ω_1
$\approx \frac{1}{|L_{DC}|}$
$\left|\frac{L_{DC}p_1p_2p_3}{\omega_3^3}\right| = \frac{-L_{DC}}{1-L_{DC}}$
$\left|\frac{L_{DC}p_1p_2p_3}{\omega^3}\right|$
$|L(\omega)|$
$|S(\omega)|$
$|A_{f\infty}(\omega)|$
$|A_f(\omega)|$
magnitude logarithmic scale
$\approx \frac{1}{|L_{DC}|}$
$|A_{f\infty}(\omega)|$
$|A_f(\omega)|$
ω_{p3}
ω_2
frequency [rad/s] logarithmic scale

Performance of negative-feedback amplifiers

Procedure for determination of dominant and non-dominant poles

1. Rank the high-frequency poles of L with increasing frequency (start with the most dominant pole and end with the highest frequency pole; here $|p_1| < |p_2| < |p_3|$)
2. Calculate ω_n for increasing order:

$$\omega_1 = |L_{DC} p_1|$$

$$\omega_2 = \sqrt{|L_{DC} p_1 p_2|}$$

$$\omega_3 = \sqrt[3]{|L_{DC} p_1 p_2 p_3|}$$

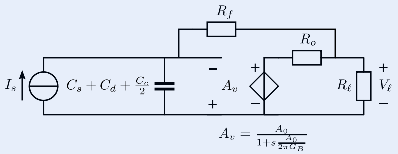
etc.

3. The order n, is the number for which ω_n has the smallest value. This number is the number of dominant poles.

Performance of negative-feedback amplifiers

Example transimpedance amplifier

Small-signal equivalent circuit:



Select A_v as loop gain reference: ideal gain = asymptotic-gain: $A_{f\infty} = -R_f$

Loop gain L calculated by network inspection:

$$L = -\frac{A_0}{1+s\frac{A_0}{2\pi G_B}} \frac{R_\ell}{R_\ell + R_o} \frac{1}{1 + s\left((R_o//R_\ell) + R_f\right)\left(C_s + C_d + \frac{C_c}{2}\right)}$$

$$L_{DC} = \frac{-A_0 R_\ell}{R_\ell + R_o}, \quad p_1 = -\frac{2\pi G_B}{A_0}, \quad p_2 = -\frac{1}{\left((R_o//R_\ell) + R_f\right)\left(C_s + C_d + \frac{C_c}{2}\right)}$$

Performance of negative-feedback amplifiers

Second order bandwidth

$$B_\omega = \sqrt{\frac{2\pi G_B R_\ell}{R_\ell + R_o} \frac{1}{\left((R_o//R_\ell) + R_f\right)\left(C_s + C_d + \frac{C_c}{2}\right)}}\ \text{[rad/s]}$$

Design equation G_B of operational amplifier:

$$G_B \geq 2\pi B_f^2 \frac{R_\ell + R_o}{R_\ell} \left((R_o//R_\ell) + R_f\right)\left(C_s + C_d + \frac{C_c}{2}\right)$$

- Unknown parameters: G_B, R_o, C_d and C_c
- C_s, R_f and R_ℓ and B_f, follow from the amplifier specification

Performance of negative-feedback amplifiers

Second order bandwidth GB show-stopper

The maximum bandwidth for any G_B is obtained if: $C_d + \frac{C_c}{2} \ll C_s$, $R_o \ll R_\ell$, and $R_o \ll R_f$, we then need:

$$G_B \geq 2\pi R_f C_s B_f^2 \text{ [Hz]}$$

Any device with a smaller value would be a *show stopper*.

Performance of negative-feedback amplifiers

Example transimpedance amplifier bandwidth design using SLiCAP

- Section 7.4.3. in the book, example 7.2
- SLiCAP execute: `~/day5/Exercises5/Exercises5.m`
- Study script and its output: `~/day5/Exercises5/transimpedance.m`

Method

1. Calculate Laplace transform of the loop gain (numeric with values for R_f, C_s and R_ℓ.
2. Determine the coefficients of the numerator polynomial and of the denominator polynomial
3. Determine the loop gain poles product from these coefficients
4. Solve G_B for $C_d = 0$, $C_c = 0$ and $R_o = 0$.

Performance of negative-feedback amplifiers

Selection of the operational amplifier and verification of the bandwidth

- Section 7.4.3. in the book, example 7.3
- SLiCAP execute: `~/day5/Exercises5/Exercises5.m`
- Study script and its output: `~/day5/Exercises5/transimpedance.m`

Method

1. Substitute component values in expression for loop gain poles product
2. Plot the magnitude and phase characteristics of all gain functions of the asymptotic-gain model:
    1. Asymptotic-gain
    2. Gain
    3. Loop gain
    4. Servo function
    5. Direct transfer

Performance of negative-feedback amplifiers

High-pass cut-off and loop gain with poles and zeros

There is more!

- AC coupling in the loop causes zeros in the loop gain at $s = 0$ and low-frequency poles
- Decoupling capacitors may introduce pole-zero pairs
- The influence of these pole-zero pairs is discussed in the textbook, section 7.7.5. through 7.4.7.
- High-pass cut-off at low frequencies, as well as pole-zero pairs in the transmission band cause droop or tilt in the step response

Frequency stability of negative-feedback amplifiers

Nyquist criterion

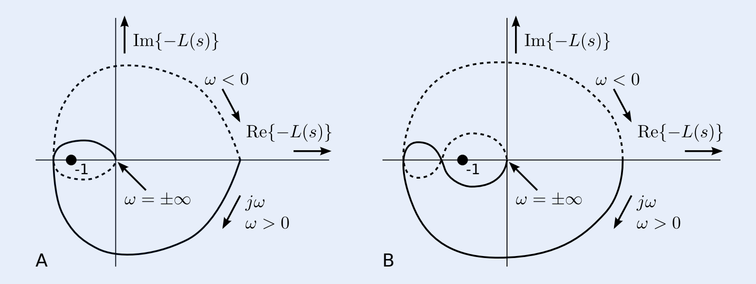
If the loop gain $-L(s)$ is plotted as a contour in the complex L plane: $\mathrm{Re}\{-L(s)\}$ versus $\mathrm{Im}\{-L(s)\}$, with $s=j\omega$ varying along the contour from 0 to $j\infty$ and from $-j\infty$ to 0, it can be shown that:

$$n = p - q$$

1. n is the number of clockwise encirclements of the point $(-1,0)$ in the complex L plane by the contour
2. p is the number of poles of the servo function $S(s)=\frac{-L(s)}{1-L(s)}$ inside the right half of the s plane
3. q is the number of poles of the loop gain $L(s)$ inside the right half of the s plane

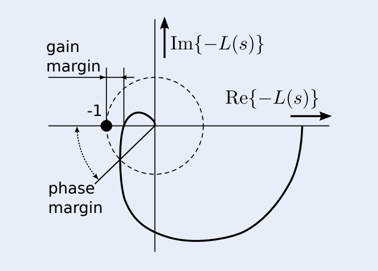
Performance of negative-feedback amplifiers

Nyquist criterion



Performance of negative-feedback amplifiers

Nyquist criterion, gain margin and phase margin



- Gain and phase margin tell us something about the stability of a loop
- In general, they have no direct relation to the frequency response of a negative feedback amplifier
- The cannot be used for the design of the filter characteristic or step response of the negative feedback amplifier

Performance of negative-feedback amplifiers

Root Locus Technique

The root locus is the path in the complex plane as it is traced out by the roots of the characteristic equation under variation of one parameter:

- Commonly used in control theory:
  - the static (DC) loop gain
- Later we will use any circuit parameter of interest as root locus variable
- SLiCAP:
  - `gainType('servo')` or `gainType('gain')`
  - `dataType('poles')` combined with parameter stepping

Performance of negative-feedback amplifiers

Root locus construction rules with DC loop gain as parameter

1. Each point satisfies $|A\beta(s)| = 1$ $\arg A\beta(s) = k2\pi,\ k = 0, 1, 2, \ldots$
2. Number of branches $=$ number of poles
3. Real or complex conjugates: symmetric with respect to real axis
4. Starts at $A\beta_{DC} = 0$: poles of $A\beta$
5. Ends at zeros, $n - m$ zeros at infinity ($n =$ n.o. poles, $m =$ n.o. zeros)
6. Parts of real axis left from odd number of poles $+$ zeros are part of root locus
7. $n - m$ asymptotes: $\theta_i = \frac{2i+1}{n-m}\pi,\ i = 0, 1, 2, \ldots$
8. Asymptotes intersect at $\sigma = \frac{\Sigma p - \Sigma z}{n-m}$
9. Poles leave real axis for $\frac{d}{ds}A\beta(s) = 0$
10. Break away equally spaced over 2π

Performance of negative-feedback amplifiers

Root locus examples

- Section 7.5.3. in the book
- SLiCAP execute: `~/day5/Exercises5/Exercises5.m`
- Study script and its output: `~/day5/Exercises5/transimpedance.m`

Performance of negative-feedback amplifiers

There is more!

- Non controllable and non observable states: Section 7.5.4.
- Analysis of differential-mode and common-mode dynamic behavior: Section 7.5.5.