Noise in Electronic Systems

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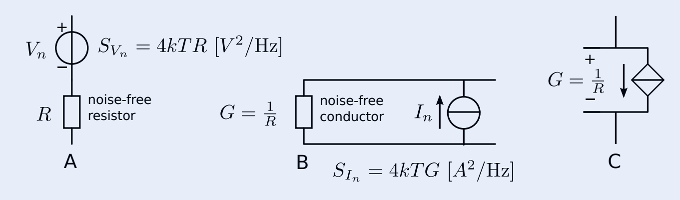
Sources of noise

Thermal noise

Noise caused by thermal movement of electrons



At constant temperature: ergodic process
Gaussian amplitude distribution and a uniform spectral density.
Noise model: noise-free resistor with voltage or current source

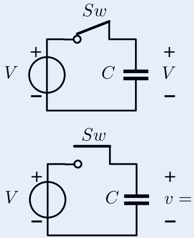
$$S_{V_n} = 4kTR \qquad [\mathrm{V}^2/\mathrm{Hz}],\ S_{I_n} = 4kTG \qquad [\mathrm{A}^2/\mathrm{Hz}]$$

$k = 1.38 \cdot 10^{-23}$ J/K (Boltzmann constant)
$T =$ K (absolute temperature)

Sources of noise

KTC noise

- Thermal noise in switched capacitor systems such as
  - Video sensor readout systems
  - Switched integrators



Due to thermal motion a noise current flows in the current loop.
When opening the switch, the charge on the capacitor is 'frozen'.
The uncertainty of the charge causes uncertainty of the voltage.
Referred to as KTC noise.

$$v = \mathrm{Gauss}(\mu = V, \sigma^2 = \frac{kT}{C})$$

Sources of noise

Shot noise

- Variations in the transport of charge carriers across a potential barrier
- Currents through voltage barriers such as PN junctions have associated shot noise currents
- Gaussian distribution density function and a uniform spectral density
- Spectral density of a shot noise current associated with a stationary (DC) junction current I_J is

$$S_{I_n} = 2qI_J \qquad [\mathrm{A}^2/\mathrm{Hz}]$$

- q is the electrical charge of the charge carrier ($1.6 \cdot 10^{-19}$ C for an electron).

Sources of noise

Excess noise

- Fluctuations in conduction mechanisms
- Found in resistors, in electrolytic capacitors and in semiconductor devices
- Gaussian amplitude distribution function
- Spectral density is inversely proportional with the frequency.

Resistors:

$$S_{V_n} = \frac{KV_R^2}{f} \qquad [\mathrm{V}^2/\mathrm{Hz}]$$

K is a material constant,
V_R is voltage across resistor.

PN junctions:

$$S_{in} = \frac{KI_J^{\alpha}}{f} \qquad [\mathrm{A}^2/\mathrm{Hz}]$$

K and α are material constants,
I_J is DC junction current.

Noise Temperature

- The noise power of a source is sometimes defined by its noise temperature.
- The noise temperature T_n is defined as:

$$T_n = \frac{P}{kB} \text{ [K]}$$

- P is the available noise power in Watts,
- B is the total bandwidth over which the noise power is measured
- k is Boltzmann constant.

Noise Bandwidth

- The equivalent noise bandwidth B_n of a system with a transfer function $H(j\omega)$:
  The bandwidth of a brickwall filter with a pass band gain equal to the maximum magnitude of $H(j\omega)$, that would produce the same output noise power as $H(j\omega)$:
  $$B_n = \frac{1}{2\pi}\int_0^\infty \left|\frac{H(j\omega)}{H_{\max}}\right|^2 d\omega \text{ [Hz]}$$

Performance parameters

Example noise bandwidth 1-st order low-pass filter

First order low-pass transfer with DC gain H_0 and -3dB bandwidth $B_{-3\text{dB}} = \frac{1}{2\pi\tau}$ [Hz]:

$$H(j\omega) = \frac{H_0}{1 + j\omega\tau}$$

The maximum value of $|H(j\omega)|$: $H_{\text{max}} = H_0$:

$$\begin{aligned} B_n &= \frac{1}{2\pi}\int_0^{\infty}\frac{1}{H_0}\left|\frac{H_0}{1 + j\omega\tau}\right|^2 d\omega \\ &= \frac{1}{2\pi\tau}\int_0^{\infty}\frac{1}{1 + \omega^2\tau^2} d\omega\tau = \frac{1}{4\tau} \\ &= \frac{\pi}{2}B_{-3\text{dB}} \end{aligned}$$

Performance parameters

Signal-to-noise ratio and dynamic range

Signal-to-noise ratio:

$$\frac{S}{N} = \frac{P_{signal}}{P_{noise}} \text{ [-]}$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \text{ [dB]}$$

Dynamic range

- Ratio of maximum signal power and noise power in the absence of signal
- Conditions for maximum signal power have to be given:
  - IMFDR: Intermodulation-free dynamic range
  - SFDR: Spurious-free dynamic range.

Performance parameters

Noise figure

Noise Figure

The noise figure is a measure for the deterioration of the signal to noise ratio by a system.
It is defined as:

$$F = \frac{S/N \text{ at the input of the system}}{S/N \text{ at the output of the system}} \text{ [-]}$$

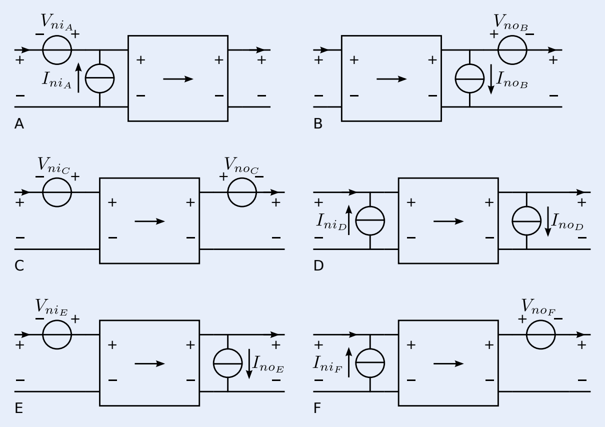
$$N = 10\log_{10} F \text{ [dB]}$$

It can only be defined if the source has a finite signal-to-noise ratio.

Noise in two-ports

Two-port representations

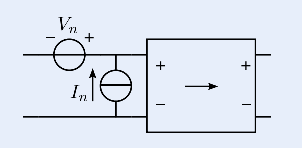
Noisy two-port represented by a noise-free two-port with two noise sources:



Equivalent input noise sources

Equivalent input noise sources

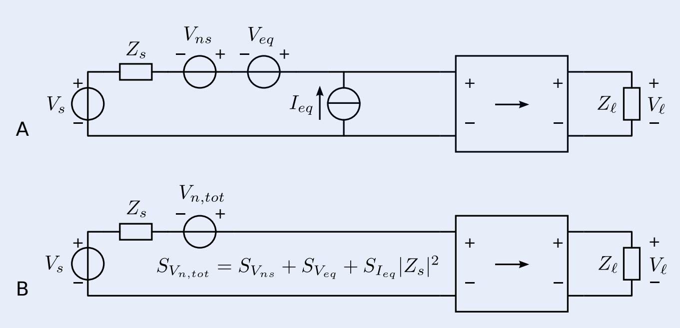
- From a design point of view, a representation with two equivalent input noise sources is preferred.



- With this model, budgets for the equivalent input noise sources can easily be derived from
  1. The source specification
  2. Budget for noise addition by the amplifier

Equivalent input noise sources

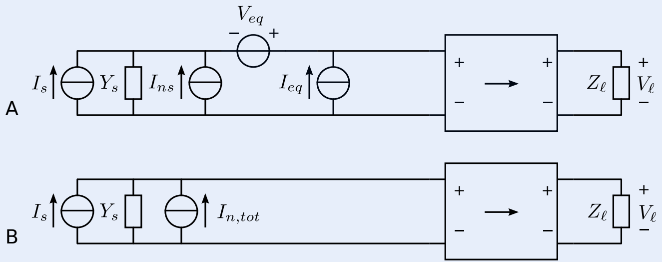
Determination of the total input voltage noise



Norton to Thévenin transformation for I_{eq} and Z_s yields a noise source of the same type as the information source.

Equivalent input noise sources

Determination of the total input current noise



$$S_{I_{n,tot}} = S_{I_{ns}} + S_{I_{eq}} + S_{V_{eq}} |Y_s|^2$$

Thévenin to Norton transformation for V_{eq} and Y_s yields a noise source of the same type as the information source.

Equivalent input noise sources

Determination of total source-referred noise

A weighting function $W(f)$ models the observer's sensitivity to the noise

The total weighted *source-referred* RMS voltage noise is:

$$V_n = \sqrt{\int_0^{\infty} S_{V_{n,tot}} \left| W(f) \right|^2 df} \text{ [V]}$$

The total weighted *source-referred* RMS current noise is:

$$I_n = \sqrt{\int_0^{\infty} S_{I_{n,tot}} \left| W(f) \right|^2 df} \text{ [A]}$$

Equivalent input noise sources

Determination of the total source-referred noise

Use source transformation techniques

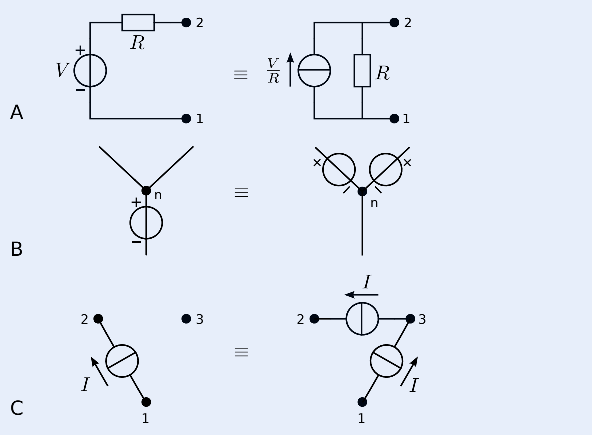
- Current-split and voltage shift theorem
- Thévenin and Norton transformation
- Two-port noise model transformations

Use Modified Nodal Analysis

1. Add noise sources to the network
2. Set signal sources to zero
3. Calculate output voltage
4. Divide the output voltage by source-output transfer

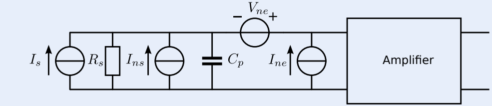
Source transformations

Thévenin-Norton, voltage-shift, current-split

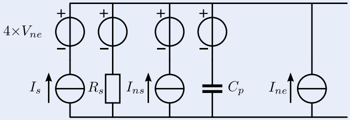


Source transformations

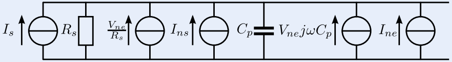
Application of Thévenin-Norton, voltage-shift



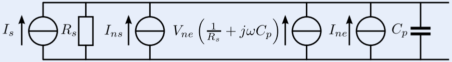
Shift V_{ne} into all the connected branches.



Replace all brances with their Norton equivalent.



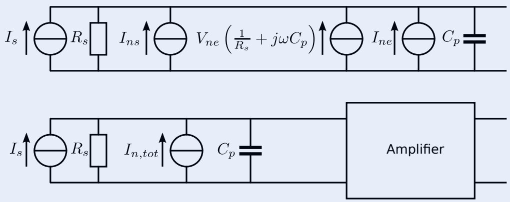
Add correlated sources.



Source transformations

Application of Thévenin-Norton, voltage-shift, results

- Source referred noise spectrum: sum of uncorrelated spectral contributions:

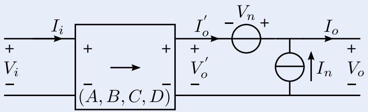


$$S_{I_{n,tot}} = S_{I_{ns}} + S_{I_{ne}} + S_{V_{ne}} \left|\frac{1}{R_s} + j\omega C_p\right|^2 = S_{I_{ns}} + S_{I_{ne}} + S_{V_{ne}} \left(\frac{1}{R_s^2} + \omega^2 C_p^2\right)$$

- Impedances in the signal path deteriorate signal to noise ratio:
  1. Addition of noise with dissipative elements
  2. Increase the influence of equivalent noise sources of subsequent stages

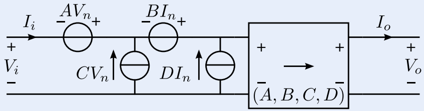
Source transformations

Two-port model transformation



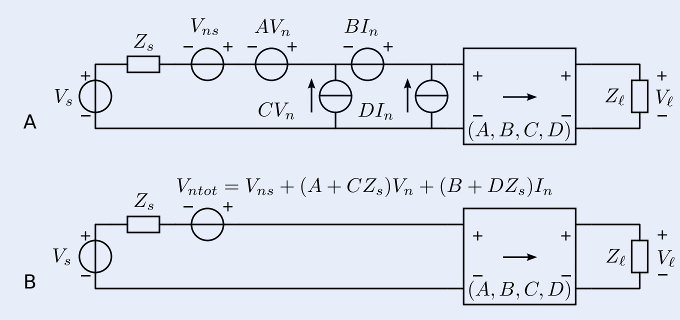
$$\left[\begin{array}{c} V_i \\ I_i \end{array}\right] = \left[\begin{array}{cc} A & B \\ C & D \end{array}\right] \left[\begin{array}{c} V_o - V_n \\ I_o - I_n \end{array}\right]$$

$$\left[\begin{array}{c} V_i + AV_n + BI_n \\ I_i + CV_n + DI_n \end{array}\right] = \left[\begin{array}{cc} A & B \\ C & D \end{array}\right] \left[\begin{array}{c} V_o \\ I_o \end{array}\right]$$



Source transformations

Two-port model transformation



Source referred noise

Total equivalent input noise

$$F = \frac{S/N \text{ at the output of the amplifier}}{S/N \text{ at the input of the amplifier}}$$

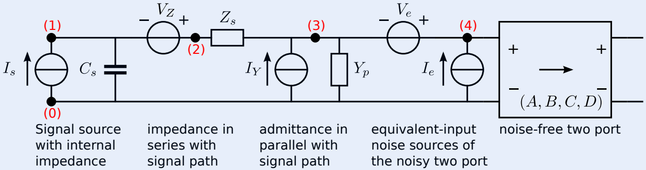
$$F = \frac{\text{(Weighted) total equivalent input noise power}}{\text{(Weighted) source noise power}}$$

Amplifier from previous slide:

$$F = \frac{\int_0^\infty \left(4kT\,\mathrm{Re}\left(Z_s\right) + \left|A + CZ_s\right|^2 S_{V_n} + \left|B + DZ_s\right|^2 S_{I_n}\right)\left|W(f)\right|^2 df}{\int_0^\infty 4kT\,\mathrm{Re}\left(Z_s\right)\left|W(f)\right|^2 df}$$

Source referred noise

Use MNA for evaluation of total source referred noise



I_s: Signal source

V_Z: Noise voltage associated with Z_s. Spectral density: $S_{V_Z} = 4kT\mathrm{Re}(Z_s), V^2/Hz$

I_Y: Noise current associated with Y_p. Spectral density: $S_{I_Y} = 4kT\mathrm{Re}(Y_p), A^2/Hz$

V_e: Equivalent-input noise voltage of the two port. Spectral density: $S_{Ve}, V^2/Hz$

I_e: Equivalent-input noise current of the two port. Spectral density: $S_{Ie}, A^2/Hz$

Source referred noise

Use MNA for evaluation of source referred noise

Procedure

1. Set up MNA equations
2. Define output variable:
   - Be sure to include the contributions of all relevant noise sources
3. Evaluate total noise associated with output variable using Cramer's rule
4. Divide that result by the transfer from the signal source to that dependent variable to obtain the source referred noise
5. Collect coefficients for each noise source
6. Calculate spectrum of source referred noise

Source transformations

Set up MNA equations

Matrix equation

$$\mathbf{I} = \mathbf{MV}$$

Include independent noise sources only

$$\begin{pmatrix} 0 \\ 0 \\ I_Y \\ I_e \\ V_Z \\ V_e \end{pmatrix} = \begin{pmatrix} j\omega C_s & 0 & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{Z_s} & -\frac{1}{Z_s} & 0 & 1 & 0 \\ 0 & -\frac{1}{Z_s} & \frac{1}{Z_s} + Y_p & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_{V_Z} \\ I_{V_e} \end{pmatrix}$$

Source transformations

Evaluate total noise at output node

Application of Cramer's rule

Output V_4, the nodal voltage at node (4)

$$V_4 = \frac{\det \mathbf{M}'}{\det \mathbf{M}}$$

$$\mathbf{M}' = \begin{pmatrix} j\omega C_s & 0 & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{Z_s} & -\frac{1}{Z_s} & 0 & 1 & 0 \\ 0 & -\frac{1}{Z_s} & \frac{1}{Z_s} + Y_p & I_Y & 0 & -1 \\ 0 & 0 & 0 & I_e & 0 & 1 \\ -1 & 1 & 0 & V_Z & 0 & 0 \\ 0 & 0 & -1 & V_e & 0 & 0 \end{pmatrix}$$

Source transformations

Evaluate total noise associated with an independent variable

Determination of transfer from signal source to independent variable V_4

$$\frac{V_4}{I_s} = \frac{\mathcal{C}_{1,4}(\mathbf{M})}{\det \mathbf{M}}$$

- Cofactor $\mathcal{C}_{1,4}(\mathbf{M}) = (-1)^{(1+4)} \det\left(\mathcal{M}_{1,4}(\mathbf{M})\right)$
- Minor matrix $\mathcal{M}_{1,4}(\mathbf{M})$: delete row 1 and column 4 from $\mathbf{M}$

$$\mathcal{M}_{1,4}(\mathbf{M}) = \det \begin{pmatrix} 0 & \frac{1}{Z_s} & -\frac{1}{Z_s} & 1 & 0 \\ 0 & -\frac{1}{Z_s} & \frac{1}{Z_s} + Y_p & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

Source transformations

Determine total source referred noise

Determination of the source referred noise I_{ns}

$$I_{ns} = \frac{V_4}{\frac{V_4}{I_s}} = \frac{\det \mathbf{M}'}{\mathcal{C}_{1,4}(\mathbf{M})}$$

- Collect terms for each noise source: this yields noise transfer functions.
- Transfer functions tell us how large an equivalent noise source (type and location of the signal source) should be to obtain the same contribution to the output noise:

$$\begin{aligned} I_{ns} = {} & I_e\left(1 + j\omega C_s Z_s\right) \\ & + I_Y\left(1 + j\omega C_s Z_s\right) \\ & + V_e\left(Y_p + j\omega C_s\left(1 + Y_p Z_s\right)\right) \\ & + j\omega C_s V_Z \end{aligned}$$

Source transformations

Determine source referred noise

Determination of the spectral density of the source referred noise I_{ns}

- Multiply the spectral density of each noise source with the square of its corresponding transfer function:

$$
\begin{aligned}
S_{I_{ns}} &= S_{I_e}\left|1 + j\omega C_s Z_s\right|^2 \\
&+ S_{I_Y}\left|1 + j\omega C_s Z_s\right|^2 \\
&+ S_{V_e}\left|Y_p + j\omega C_s\left(1 + Y_p Z_s\right)\right|^2 \\
&+ S_{V_Z}\omega^2 C_s^2
\end{aligned}
$$