Principle of Amplification

Application of Nonlinear Devices and Power Sources

Anton J.M. Montagne

Perscitec B.V.

September 25, 2017

Introduction

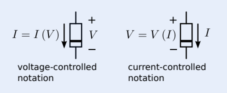
Aim of this lecture

1. Understand the amplification mechanism embodied in 'active devices'
   - Vacuum tubes, BJTs, FETs and MOSFETs are passive devices
   - Combinations of the above devices and power sources may result in *active* behavior
   - Active network element delivers power to passive network element
     - Passive element: product of branch voltage and branch current is positive
     - Active element: product of branch voltage and branch current is negative
2. Understand the concept of biasing
   - Available power gain of biased stage may exceed unity: load signal power mainly provided by bias sources
   - Device properties and quiescent operating point determine amplifying capabilities
   - Conceptual biasing with ideal voltage and current sources only
   - Design of biasing circuitry separated from design oif signal processing

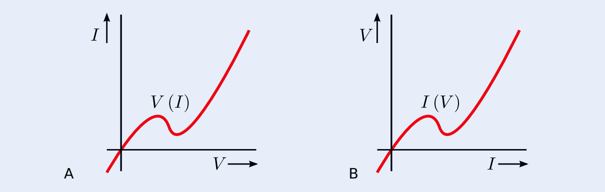
Two-terminal resistive elements

V-I Characteristics

Symbol



Voltage and current-controlled notation

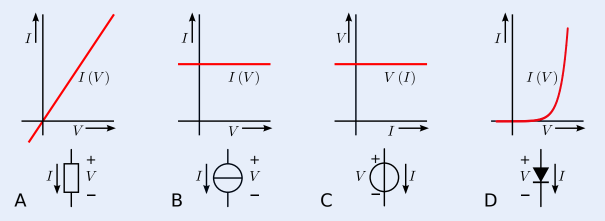


Voltage-controlled and current-controlled representation of the $v - i$ characteristics of a passive two-terminal resistive element.

Two-terminal resistive elements

Examples

element	parameters	current-controlled	voltage controlled
linear resistor	R	$V(I) = I\mathrm{R}$	$I(V) = \frac{V}{\mathrm{R}}$
voltage source	V	$V(I) = \mathrm{V}$	$I(V)$: undefined
current source	I	$V(I)$: undefined	$I(V) = \mathrm{I}$
ideal diode	$\mathrm{IS},\ \mathrm{UT}$	$V(I) : \mathrm{VT}\ln\left(1+\frac{I}{\mathrm{IS}}\right)$ valid for $I > -\mathrm{IS}$	$I(V) : \mathrm{IS}\exp\left(\frac{V}{\mathrm{UT}}\right)$



Two-terminal resistive elements

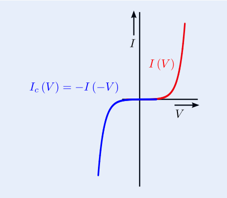
Complementary devices

$v - i$ Characteristics of complementary devices:

$$R_1 : V = V(I) \text{ or } I = I(V)$$
$$R_2 : V = V_c(I) \text{ or } I = I_c(V)$$

R_1 and R_2 are said to be complementary if:

$$V_c(i) = -V(-I) \text{or } I_c(V) = -I(-V)$$



Two-terminal resistive elements

Operating point and biasing

Selection of operating point (V_Q, I_Q)

v = voltage deviation from V_Q

i = current deviation from I_Q

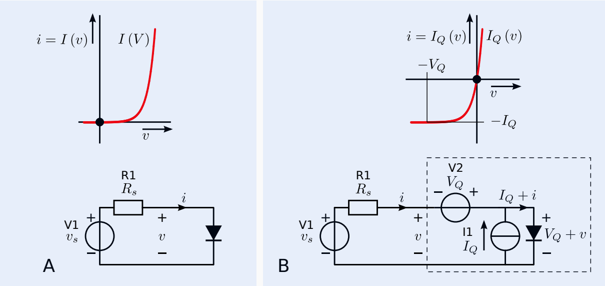
$$I_Q + i = I\left(V_Q + v\right); \text{ voltage-controlled representation}$$

$$V_Q + v = V\left(I_Q + i\right); \text{ current-controlled representation}$$

Two-terminal resistive elements

Operating point and biasing

Fix operating point with a bias voltage and a bias current source:



Two-terminal resistive elements

Dependent and independent bias sources and biasing errors

Dependent and independent bias sources

- Either the bias voltage or the bias current can be selected by design
- The other follows from the selected one and the devices $V - I$ characteristic at the operating temperature

Biasing errors

- $V - I$ characteristic of the device is not fully known
- $V - I$ characteristic of the device is subjective to temperature changes
- Implementation of biasing sources not without errors

If biasing errors become too large, error-reduction techniques have to be applied. This will be discussed at a later stage.

Two-terminal resistive elements

Linearization in the operating point

The $v - i$ relation of the biased non-linear device can be written as:

$$I = I_Q(V) \text{ with: } I_Q(V) = I(V_Q + v)$$
$$V = V_Q(I) \text{ with: } V_Q(\tilde{\imath}) = V(I_Q + i)$$

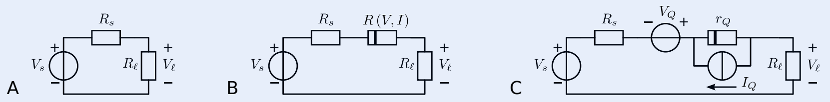
Small-signal behavior:

- For small deviations from the operating point, the characteristics can be considered linear.
- The small-signal characteristics in the operating point Q are:

$$i = g.v; \qquad g = \left.\frac{I_Q(v)}{dv}\right|_{v=0}$$
$$v = r.i; \qquad r = \left.\frac{V_Q(i)}{di}\right|_{i=0}$$

Two-terminal resistive elements

Available power gain



- Available power gain = ratio of available power at the output port of the amplifier and the available power of the source.
    - The available power of the source

$$P_{\mathrm{av}} = \frac{V^2}{4R_s}$$

    - Amplifier is biased two-terminal device, available power:

$$P'_{\mathrm{av}} = \frac{V^2}{4\left(R_s + r_Q\right)}$$

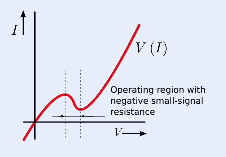
Two-terminal resistive elements

Available power gain > 1: negative small-signal resistance

- The available power gain: ratio of P'_{av} and P_{av}:

$$G_P = \frac{P'_{av}}{P_{av}} = \frac{R_s}{R_s + r_Q}$$

  - Exceeds unity if the small-signal resistance r_Q of the two-terminal device in the operating point Q has a negative value.
- The tunnel diode is an example of a two-terminal device that has a negative small-signal resistance in a certain operating region.



Nonlinear multi-terminal resistive elements

Multi-terminal, complementary, resistive devices

Complementary multi-terminal resistive devices $\mathbf{M_1}$ and $\mathbf{M_2}$:

$$M_1 : f_{1...n}\left(V_1...V_n, I_1...I_n\right) = 0$$
$$M_2 : g_{1...n}\left(V_1...V_n, I_1...I_n\right) = 0$$

These elements are said to be complementary if:

$$g_{1...n}\left(V_1...V_n, I_1...I_n\right) = -f_{1...n}\left(-V_1... - V_n, -I_1... - I_n\right)$$

Nonlinear multi-terminal resistive elements

Two-port representations

Nonlinear resistive two-port representations:

Voltage-controlled representation:	$I_i = I_i\left(V_i, V_o\right)$	$I_o = I_o\left(V_i, V_o\right)$
Current-controlled representation:	$V_i = V_i\left(I_i, I_o\right)$	$V_o = V_o\left(I_i, I_o\right)$
Hybrid 1 representation:	$I_i = I_i\left(V_i, I_o\right)$	$V_o = V_o\left(V_i, I_o\right)$
Hybrid 2 representation:	$V_i = V_i\left(I_i, V_o\right)$	$I_o = I_o\left(I_i, V_o\right)$
Transfer 1:	$I_o = i_o\left(V_i, I_i\right)$	$V_o = V_o\left(V_i, I_i\right)$
Transfer 2:	$V_i = V_i\left(v_o, I_o\right)$	$I_i = I_i\left(V_o, I_o\right)$

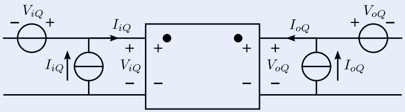
Nonlinear multi-terminal resistive elements

Operating point and biasing

- Two-port equations in voltage-controlled notation:

$$I_i = I_i\left(V_i, V_o\right)$$
$$I_o = I_o\left(V_i, V_o\right)$$

- Selection of operating point; relation between bias quantities given by device equations:



- Two-port equations of biased device in voltage-controlled notation:

$$i_i = I_{iQ}\left(v_i, v_o\right)$$
$$i_o = I_{oQ}\left(v_i, v_o\right)$$

Nonlinear multi-terminal resistive elements

Available power gain of linearized biased nonlinear two-port

Transmission parameter representation:

$$\begin{pmatrix} V_i \\ I_i \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_o \\ I_o \end{pmatrix}$$

$$A = \left. \frac{\partial V_{iQ}\left(v_o, i_o\right)}{\partial \tilde{v}_o} \right|_{i_o=0}, \quad B = - \left. \frac{\partial V_{iQ}\left(v_o, i_o\right)}{\partial i_o} \right|_{v_o=0}$$

$$C = \left. \frac{\partial I_{iQ}\left(v_o, i_o\right)}{\partial v_o} \right|_{i_o=0}, \quad D = - \left. \frac{\partial I_{iQ}\left(v_o, i_o\right)}{\partial i_o} \right|_{v_o=0}$$

Available power gain:

$$G_P = \frac{1}{AD + AB/R_s + BC + CDR_s}$$