

Device modeling

Modeling of 'Active Devices'

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September 25, 2017

Introduction

Aim of this lecture

- ① Understand the basic operation of amplifying devices
 - MOSFETs
 - BJT_s
 - JFET_s
- ② Be able to model performance aspects relevant for amplifier design
 - Static behavior
 - Dynamic behavior
 - Noise behavior
 - Operating regions

Introduction

Analysis types and device models

① Symbolic analysis for setting up design equations

- Parameterized small-signal models
 - Behavior linearized in operating point
- Model parameters depend on
 - Device type and characteristic
 - Device geometry
 - Operating point

② Numeric analysis for design verification

- Static behavior (DC analysis model)
- Small-signal dynamic behavior (AC analysis model)
- Large signal dynamic behavior (Transient analysis model)
- Small-signal dynamic noise behavior (Noise model)

Introduction

All models are wrong

- Simulators give high-resolution output (many digits)
- Predictability of the circuit behavior can still be low:
 - Have the relevant physical effects been modeled (accurately)?
 - Have the model parameters been extracted from the measurement data (correctly)?
 - Have the correct numeric algorithms been implemented (accurately)
 - Have statistical deviations been described (correctly)?

MOS transistors

Technology

- Field effect devices with their gate isolated from the channel by SiO_2
 - The Metal Oxide Semiconductor Transistor was invented by Kahng and Dawon in 1959
 - The first CMOS circuit has been reported in 1963 by Wanlass and Sah
 - Aluminium and polysilicon are commonly used as gate materials
- Depending on their application MOS transistors are fabricated in different ways:
 - Standard IC CMOS lateral devices with a symmetrical structure
 - Modern IC Fin FETs with a three dimensional gate structure
 - Lateral or vertical high-voltage MOS transistors of which the low-voltage gate-source structure differs from the high-voltage gate-drain structure

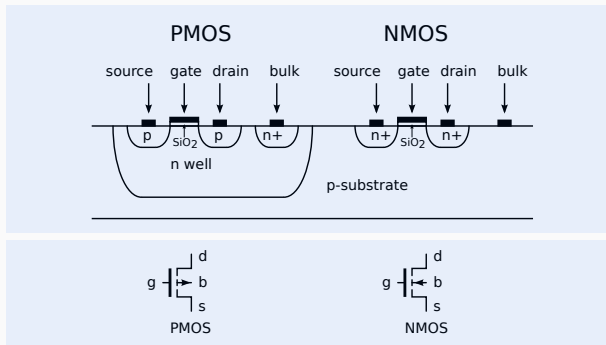
MOS transistors

Application

- MOS transistors found their first application in digital circuits
- High-volume analog ICs and mixed signal ICs nowadays predominantly fabricated in CMOS technology
- Power MOS transistors are available in high-voltage IC processes and as discrete devices, mostly used as power switches in switched regulators and switched amplifiers

MOS transistors

Structure



MOS transistors

Operation: weak inversion

- Assume:
 - Source and the bulk (substrate) of an NMOS have been connected to a reference potential (0V)
 - Drain has been connected to a fairly large positive voltage V_{DS} .
- Brief description of $I_{DS}(V_{GS})$ in weak inversion
 - $V_{GS} = 0$: bulk *surface potential* Ψ_s defined by the built-in junction voltage and the oxide capacitance
 - $V_{GS} > 0$: Ψ_s almost linearly increases with V_{GS} , source starts injecting electrons in the p-region
 - These electrons start to form an n-type channel under the gate and we speak of *weak inversion*
 - These electrons reach the accelerating electrical field of the drain depletion layer due to diffusion and are collected at the drain terminal
 - The $I_{DS}(V_{GS})$ characteristic shows an exponential relationship similar as the $I_C(V_{BE})$ relationship of a lateral bipolar NPN transistor

MOS transistors

Operation, strong inversion

- Brief description of $I_{DS}(V_{GS})$ in strong inversion
 - When $V_{GS} > V_T$ the channel extends further under the gate and the Ψ_s does not longer increase significantly with V_{GS}
 - I_{DS} now increases quadratically with $V_{GS} - V_T$
 - The device now operates in *strong inversion*
 - The *threshold voltage* V_T is the gate-source voltage at the transition from weak inversion to strong inversion

MOS transistors

Operation, saturation region and linear region

- If the *effective gate-source* voltage $V_{GS} - V_T$ is below V_{DS} the channel does not extend to the drain.
- In this *pinch-off* or *saturation* region the drain-source current shows a minor dependence on V_{DS} due to the *channel length modulation*
- If V_{GS} increases to V_T above V_{DS} (this is equivalent to: $V_{GD} > V_T$) the channel extends from the source to the drain:
 - I_{DS} becomes strongly dependent on V_{DS}
 - The transistor acts as a controlled resistor and we speak of the *linear operating* region

MOS transistors

Operation, small-geometry effects

With shrinking dimensions in CMOS IC processes, small geometry effects have to be taken into account:

- ① The influence of the vertical component of the electrical field in the channel on the charge carrier mobility cannot longer be ignored: VFMR
- ② At high lateral field strengths the velocity of the charge carriers saturates.
- ③ The electrical field in the gate increases with shrinking oxide thickness. This causes a slow change of charge storage in the oxide over time and results in a slow change of V_T over time which may limit the operational life time of CMOS ICs
- ④ The drain depletion region reaches far and deep under the channel and reduces the channel to bulk depletion capacitance under the channel. This causes a better coupling between V_{GS} and Ψ_s . This *Drain Induced Barrier Lowering* (DIBL) effect causes V_T to decrease with V_{DS} and I_{DS} to increase with V_{DS}

MOSFET modeling

Introduction, first generation models

- ① Level 1 model is usually referred to the model presented by Shichman and Hodges:
 - relatively simple first generation MOS model for thick oxide long and wide channel MOSFETs operating in strong inversion
- ② Level 2 model is also a first generation model.
 - based on the Level 1 with corrections that account for small geometry effects
- ③ Level 3 model
 - based on Level 1 model, added equations for small geometry effects have a more empirical character and are more simple than those of the Level 2 model. Also includes basic modeling of weak inversion operation

MOSFET modeling

Introduction, second generation models

- ① BSIM (Berkeley Short-Channel IGFET Model) is a second generation MOSFET model
 - focus on fast and robust simulation rather than on a physical basis
- ② HSPICE Level 28 model is a second generation model based on the BSIM model. Its improvements strongly facilitated analog CMOS circuit design. It is a proprietary model developed by Meta-Software.
- ③ BSIM2 model is also a second generation MOSFET model.
 - Improvement of the BSIM model developed by the university of Berkeley.

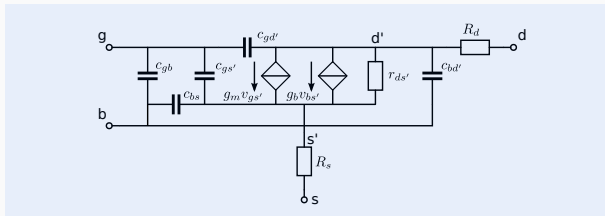
MOSFET modeling

Introduction, third generation models

- 1 Third generation BSIM3 model includes the geometry parameters in the intrinsic model.
- 2 MOS Model 9 developed at Philips Laboratories has been made generally available still distinguishes an intrinsic and extrinsic structure, but has improved mathematical modeling.
- 3 EKV Model has a physical base and takes the bulk node as reference. Source and drain are treated equally yielding a symmetrical model with smooth analytical equations for all operating regions.
- 4 BSIM4 is suited for analog CMOS design in technology nodes below 100nm.

MOSFET modeling

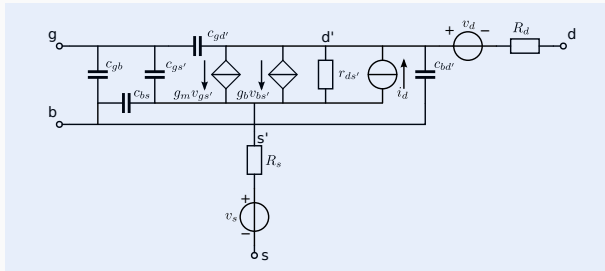
Level 1 small-signal model



- Intrinsic MOS transistor with:
 - g_m and g_b
 - $C_{gs} = \frac{2}{3}C_{ox}$, $C_{gd} = 0$ in saturation region and $C_{gs}, C_{gd} = \frac{1}{2}C_{ox}$ in linear region.
- Junction capacitances
- Bulk resistors

MOSFET modeling

Level 1 small-signal noise model



- Small-signal model with added noise sources:
 - Gate shot noise i_g and channel thermal noise i_{ds}
 - Noise associated with bulk resistors

MOSFET modeling

Level 1 small-signal noise model

Spectral densities S_{vs} and S_{vd} : voltage noise of the bulk resistors R_s and R_d :

$$S_{vs} = 4kTR_s$$

$$S_{vd} = 4kTR_d$$

Noise related to the channel current I_{dsQ} :

$$S_{id} = 4kTg_m \frac{2}{3} + \frac{KF}{f} I_{dsQ}^{AF}$$

$$S_{id} = 4kTg_m \frac{2}{3} \left(1 + \frac{f_\ell}{f} \right)$$

$$f_\ell = \frac{3KF}{8kTg_m} I_{dsQ}^{AF}$$

f_ℓ is the corner frequency for the $\frac{1}{f}$ noise.

MOSFET modeling

Level 1 small-signal model parameters

par.	cut-off.	linear region	saturation region
C_{gs}	$cgso.w + cgbo.l +$ $+ \frac{W.L}{TOX} \epsilon_{ox} (\text{max. value})$	$cgso.w + cgbo.l +$ $+ \frac{1}{2} \frac{W.L}{TOX} \epsilon_{ox} (\text{max. value})$	$cgso.w + cgbo.l +$ $+ \frac{2}{3} \frac{W.L}{TOX} \epsilon_{ox} (\text{max. value})$
C_{gd}	$cgdo.w$	$cgdo.w +$ $+ \frac{1}{2} \frac{W.L}{TOX} \epsilon_{ox} (\text{max. value})$	$cgdo.w$
C_{ds}	$cj.ad + cjsw.pd \text{ or } cbd$	$cj.ad + cjsw.pd \text{ or } cbd$	$cj.ad + cjsw.pd \text{ or } cbd$
g_m	0	$\frac{W}{L} . kp V_{DS}$	$\frac{W}{L} . kp (V_{DS} - V_{TO}) =$ $= \sqrt{2 \frac{W}{L} . KP I_{DS}}$
r_{ds}	∞	$\left[\frac{W}{L} . KP (V_{GS} - V_{TO}) \right]^{-1}$ voltage controlled resistor approximation ($V_{DS} = 0$)	$1 / (\text{lambda } I_{DS})$ ($\text{lambda} \ll 1$)

MOSFET capacitance modeling

Meyer Model

Linear region:

$$c_{gs'} = \frac{2}{3} W.L C_{ox} \left(1 - \frac{(V_{gd'} - V_t)^2}{(V_{gs'} - V_t + V_{gd'} - V_t)^2} \right) + CGSO.W$$

$$c_{gd'} = \frac{2}{3} W.L C_{ox} \left(1 - \frac{(V_{gs'} - V_t)^2}{(V_{gs'} - V_t + V_{gd'} - V_t)^2} \right) + CGDO.W$$

$$c_{gb} = 0$$

Saturation region:

$$c_{gs'} = \frac{2}{3} W.L C_{ox} + CGSO.W, c_{gd'} = CGDO.W, c_{gb} = 0$$

Cut-off region we have:

$$c_{gs'} = CGSO.W, c_{gd'} = CGDO.W, c_{gb} = W.L C_{ox} + CGBO.L \text{ (max. } V_{GB} = 0)$$

MOSFET capacitance modeling

Ward-Dutton capacitance model

- Ward and Dutton use a capacitance matrix that relates the four dynamic terminal currents of the intrinsic transistor to the terminal voltages:

$$\begin{pmatrix} i_g \\ i_d \\ i_s \\ i_b \end{pmatrix} = s \begin{pmatrix} C_{GG} & -C_{GD} & -C_{GS} & -C_{GB} \\ -C_{DG} & C_{DD} & -C_{DS} & -C_{DB} \\ -C_{SG} & -C_{SD} & C_{SS} & -C_{SB} \\ -C_{BG} & -C_{BD} & -C_{BS} & C_{BB} \end{pmatrix} \begin{pmatrix} v_g \\ v_d \\ v_s \\ v_b \end{pmatrix}$$

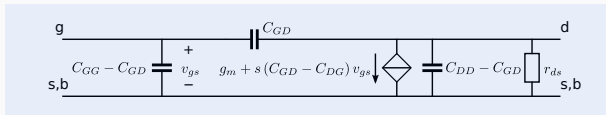
where i_g , i_d , i_s and i_b are the terminal currents of the intrinsic transistor and v_g , v_d , v_s and v_b the corresponding intrinsic terminal voltages.

MOSFET capacitance modeling

Ward-Dutton capacitance model and hybrid-pi model

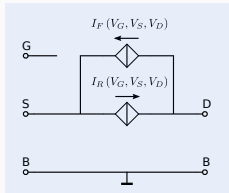
- If source and bulk are connected together and taken as reference, we have $v_s = 0$ and $v_b = 0$ and the matrix simplifies to:

$$\begin{pmatrix} i_g \\ i_d \end{pmatrix} = s \begin{pmatrix} C_{GG} & -C_{GD} \\ (C_{GD} - C_{DG}) & C_{DD} \end{pmatrix} \begin{pmatrix} v_{gs} \\ v_{ds} \end{pmatrix}$$



MOSFET modeling

EKV model summary



- Model developed by Ernst and Vittoz
- Physical basis
- Voltage-controlled expression for the channel current $I_{DS}(V_{SB}, V_{GB}, V_{DB})$ for all regions of operation
- Application of Inversion Coefficient for design described by Binkley

MOSFET modeling

EKV model: technology current

Binkley defines the technology current I_0 as:

$$I_0 = 2n\mu_0 C'_{OX} U_T^2 \text{ [A]}$$

substrate factor n :

$$n \approx 1 + \frac{C'_{DEP}}{C'_{OX}} \text{ [-]}$$

oxide capacitance per unit of area C'_{OX} :

$$C'_{OX} = \frac{\epsilon_o \epsilon_r}{t_{ox}} \text{ [Fm}^{-2}\text{]}$$

ϵ_r is the relative permittivity of SiO_2 , t_{ox} is the thickness of the gate oxide and U_T is the thermal voltage. C'_{DEP} is the surface depletion capacitance.

MOSFET modeling

EKV model: transconductance factor

The transconductance factor β_{sq} is defined as:

$$\beta_{sq} = \mu_0 C'_{OX} [AV^{-2}m^{-2}]$$

μ_0 is the low-field channel carrier mobility in $[m^2V^{-1}s^{-1}]$.

The technology current can be expressed in β_{sq} as:

$$I_0 = 2n\beta_{sq}U_T^2 [A]$$

MOSFET modeling

EKV model: weak inversion

In weak inversion the forward drain-source current I_F shows an exponential relation with the gate voltage:

$$I_{F,R} = I_0 \exp \left(\frac{\frac{V_G - V_{T0}}{n} - V_{S,D}}{U_T} \right) \text{ [A]}$$

The voltage V_{T0} is the *equilibrium threshold voltage*. The pinch-off voltage V_P is defined as:

$$V_P = \frac{V_G - V_{T0}}{n} \text{ [V]}$$

MOSFET modeling

EKV model: strong inversion

In strong inversion the currents depend quadratically on the driving voltage:

$$I_{F,R} = \frac{W}{L} \frac{\beta_{sq}}{2n} (V_G - V_{T0} - nV_{S,D})^2 \text{ [A]}$$

Where W and L are the effective width and length of the channel, respectively.

MOSFET modeling

EKV model: weak inversion to strong inversion

With the aid of a transition function $F(x)$, the expressions for weak inversion and strong inversion can be combined into one:

$$\begin{aligned} F(x) &= \left(\ln \left(1 + \exp \left(\frac{x}{2} \right) \right) \right)^2 [-] \\ &= \begin{cases} \exp(x) & \text{if } x \ll 0 \\ \left(\frac{x}{2} \right)^2 & \text{if } x \gg 0 \end{cases} \end{aligned}$$

This function returns the forward and the reverse *inversion coefficients*, IC_F and IC_R :

$$IC_{F,R} = F \left(\frac{V_G - V_{T0} - nV_{S,D}}{nU_T} \right) [-]$$

MOSFET modeling

EKV model: inversion coefficients

- The inversion coefficients are a measure for the level of inversion at which the transistor operates.
- An inversion coefficient much smaller than unity indicates weak inversion.
- An inversion coefficient much larger than unity indicates operation in strong inversion.
- Between weak and strong inversion we speak of moderate inversion.
- The actual forward and reverse current can be calculated from their respective inversion coefficients as:

$$I_{F,R} = I_0 \frac{W}{L} IC_{F,R} \text{ [A]}$$

- Or, alternatively:

$$I_{F,R} = 2n\beta_{sq} U_T^2 \frac{W}{L} IC_{F,R} \text{ [A]}$$

MOSFET modeling

EKV model: VFMR

- VFMR is modeled as reduction of the transconductance factor β_{sq} due to the gate-source voltage.
- A simple mobility reduction model uses the vertical field mobility reduction factor θ [V^{-1}].

$$\beta'_{sq} = \frac{\beta_{sq}}{1 + \theta V_P} [AV^{-2}m^{-2}]$$

- With this expression β'_{sq} increases below threshold and has a singularity at $\theta V_P = -1$.
- This can be corrected by softly clipping the VFMR effect below threshold:

$$\beta'_{sq} = \frac{\beta_{sq}}{1 + 2\theta U_T \sqrt{IC_F}} [AV^{-2}m^{-2}]$$

MOSFET modeling

EKV model: CLM

- The channel length modulation can be modeled with an Early voltage. The Early voltage V_A is approximately proportional with the length of the device:

$$V_A = V_{AL} L [-]$$

Where V_{AL} [Vm^{-1}] is the Early voltage per unit of length.

- After including the CLM the expression of the drain-source current I_{DS} changes to:

$$I_{DS} = (I_F - I_R) \left(1 + \frac{V_S - V_D}{V_{AL} L} \right) [\text{A}]$$

For reverse operation V_S and V_D should be swapped.

MOSFET modeling

EKV model: Velocity Saturation

- Binkley describes the modeling of velocity saturation for the saturation region through introduction of a second term in β_{sq} :

$$\beta'_{sq} = \frac{\beta_{sq}}{1 + \left(\theta + \frac{1}{E_{CRIT} L} \right) 2U_T \sqrt{IC_F}} \text{ [AV}^{-2}\text{m}^{-2}\text{]}$$

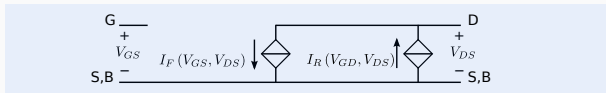
Where E_{CRIT} [Vm^{-1}] is the value of the lateral field at which velocity saturation occurs.

- At a later stage we will use the critical inversion coefficient IC_{CRIT} : the inversion coefficient at which the reduction of the mobility due to VFMR and velocity sets in:

$$IC_{CRIT} \approx \frac{1}{\left(4nU_T \left(\theta + \frac{1}{L E_C} \right) \right)^2} [-]$$

MOSFET modeling

EKV model: Static device characteristics



- In the saturation region I_R can be ignored. If we discard velocity saturation and CLM and take $V_S = 0$ we obtain the Level 1 model equation for the saturation region:

$$I_{DS} = \frac{W}{L} \frac{\beta_{sq}}{2n} (V_{GS} - V_{T0})^2 \text{ [A]}$$

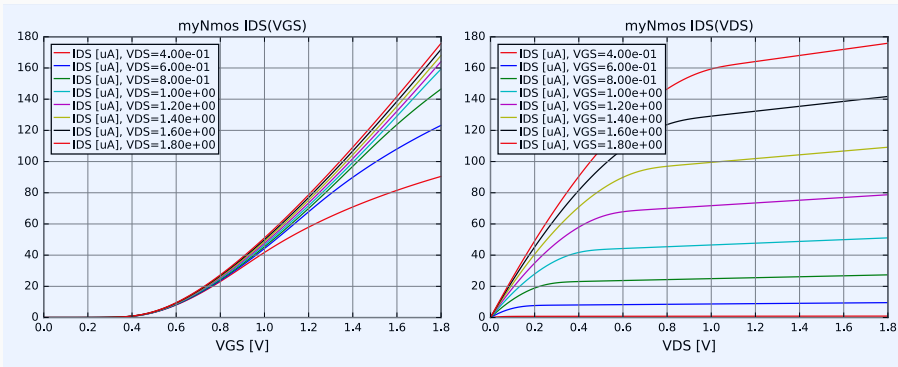
- In the linear region we cannot ignore the reverse current component. With $V_S = 0$ we obtain the Level 1 model equation for the linear region:

$$I_{DS} = \frac{W}{L} \frac{\beta_{sq}}{n} V_D \left(V_{GS} - V_{T0} - \frac{1}{2} V_{DS} \right) \text{ [A]}$$

MOSFET modeling

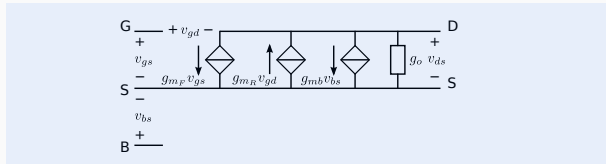
EKV model: Static device characteristics

NMOS transistor myNMOS ($W = 220\text{nm}$, $L = 180\text{nm}$) calculated according to the above (simplified) model with parameters of a standard CMOS18 process as listed by Binkley.



MOSFET modeling

EKV model: Small-signal model transconductance

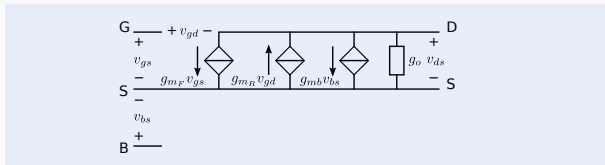


- Binkley saturation region:

$$g_{mF} = \frac{I_F \left(1 + \frac{V_S - V_D}{V_{AL} L} \right)}{n U_T \sqrt{I_{CF} \left(1 + \frac{I_{CF}}{I_{CRIT}} \right)} + 0.5 \sqrt{I_{CF} \left(1 + \frac{I_{CF}}{I_{CRIT}} \right)} + 1} \quad [AV^{-1}]$$

MOSFET modeling

EKV model: Small-signal model transconductance



- Reverse transconductance approximation (incorrect modeling of velocity saturation)

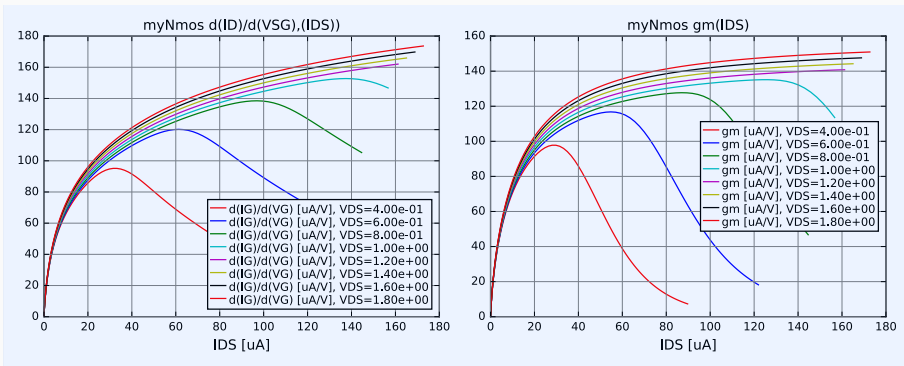
$$g_{mR} = \frac{I_R}{nU_T \sqrt{I_{CR} \left(1 + \frac{I_{CR}}{I_{CRIT}}\right)} + 0.5 \sqrt{I_{CR} \left(1 + \frac{I_{CR}}{I_{CRIT}}\right)} + 1} \quad [AV^{-1}]$$

- The total transconductance is found as the difference between the two:

$$g_m = g_{mF} - g_{mR} \quad [AV^{-1}]$$

MOSFET modeling

EKV model: Small-signal model transconductance

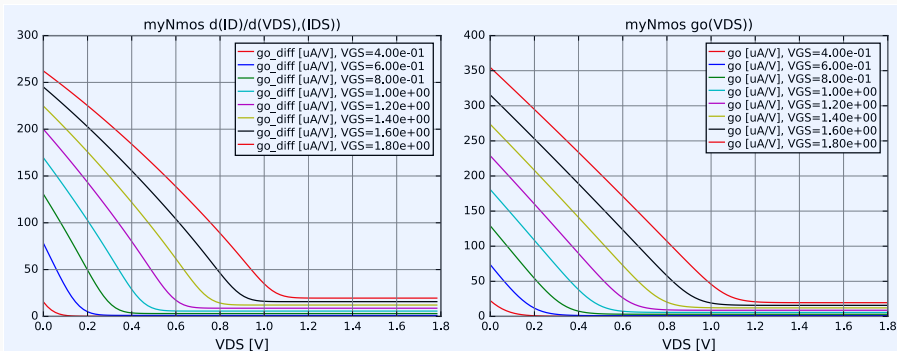


MOSFET modeling

EKV model: Output conductance

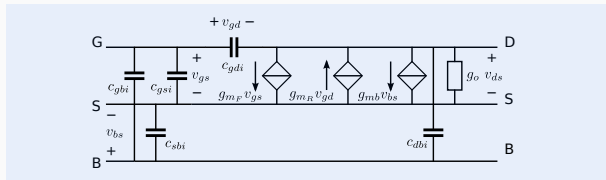
- Approximation (no DIBL)

$$g_{ds} = \frac{V_{DS} + V_{AL}L}{I_{DS}} + \frac{\beta_{sq}U_T}{1 + 2\theta U_T\sqrt{I_{CF}}} \frac{W}{L} 2\sqrt{I_{CR}} \quad (1)$$



MOSFET modeling

EKV model: Small-signal intrinsic capacitances



The intrinsic capacitances can be expressed as a part of the total oxide capacitance C_{OX} :

$$C_{OX} = WLC'_{OX} \text{ [F]}$$

The SPICE EKV2.6 model uses two parameters for calculation of these relative parts:

$$x_f = \sqrt{\frac{1}{4} + IC_F} \text{ [-]} , x_r = \sqrt{\frac{1}{4} + IC_R} \text{ [-]}$$

MOSFET modeling

EKV model: Small-signal intrinsic capacitances

The intrinsic capacitances are:

$$c_{gsi} = \frac{2}{3} \left(1 - \frac{x_r^2 + x_r + \frac{1}{2}x_f}{(x_r + x_f)^2} \right) C_{OX} \text{ [F]}$$

$$c_{gdi} = \frac{2}{3} \left(1 - \frac{x_f^2 + x_f + \frac{1}{2}x_r}{(x_r + x_f)^2} \right) C_{OX} \text{ [F]}$$

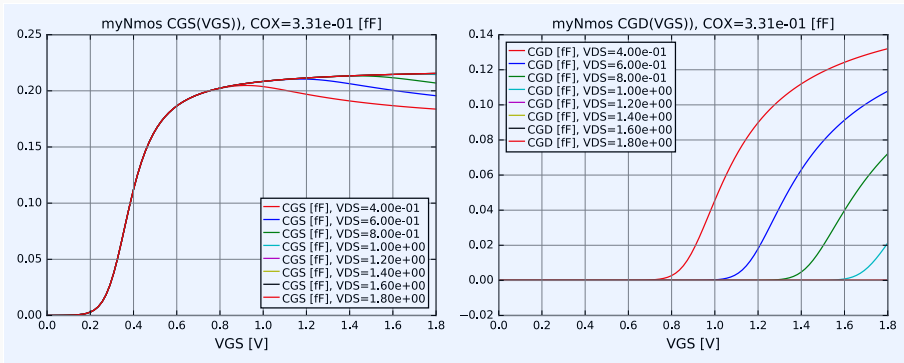
$$c_{gbi} = \frac{n-1}{n} (C_{OX} - c_{gsi} - c_{gdi}) \text{ [F]}$$

$$c_{sbi} = (n-1) c_{gsi} \text{ [F]}$$

$$c_{sdi} = (n-1) c_{gdi} \text{ [F]}$$

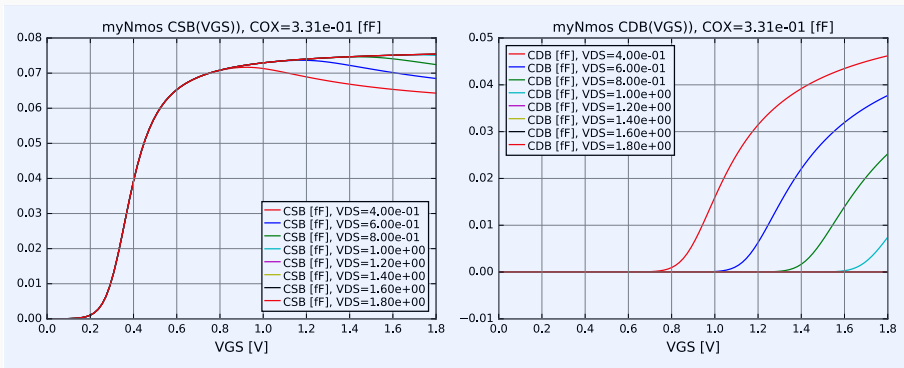
MOSFET modeling

EKV model: Small-signal intrinsic capacitances



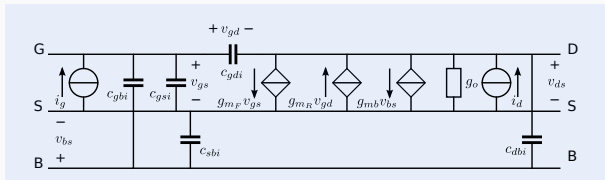
MOSFET modeling

EKV model: Small-signal intrinsic capacitances



MOSFET modeling

EKV model: Intrinsic noise model



MOSFET modeling

EKV model: Intrinsic noise model

- The spectral density S_{i_g} of the shot noise i_g associated with the gate leakage current I_G

$$S_{i_g} = 2qI_G \left(1 + \frac{f_{lg}}{f} \right) [\text{A}^2\text{Hz}^{-1}]$$

- The spectral density of i_d differs for the linear and the saturation region.
- In the linear region ($V_{DS} < V_{DS,sat}$) the spectral density S_{id} is that of a resistor with resistance $\frac{1}{g_o}$:

$$S_{id,lin} = 4kTg_o [\text{A}^2\text{Hz}^{-1}]$$

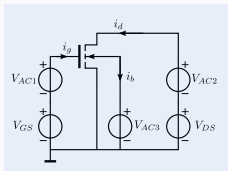
- In the saturation region, the spectral density S_{id} can be written as:

$$S_{id,sat} = 4kTn\Gamma g_m [\text{A}^2\text{Hz}^{-1}]$$

- Where $\Gamma = \frac{2}{3}$ in strong inversion ...

Determination of small-signal parameters

Simulation test bench



- ① Bias the device in the required operating point with V_{GS} and V_{DS}
- ② Add an AC signal (AC 1 0) to V_{AC1} only
- ③ Perform an AC analysis over the frequency range of interest
- ④ Obtain the following small-signal parameters:

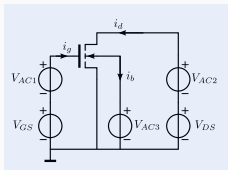
$$g_m = \text{Re} \{ I(V_{AC2}) \}$$

$$C_{gg} = C_{iss} = - \frac{\text{Im} \{ I(V_{AC1}) \}}{2\pi f}$$

$$C_{gb} = \frac{\text{Im} \{ I(V_{AC3}) \}}{2\pi f}$$

Determination of small-signal parameters

Simulation test bench



- ① Now add the AC signal (AC 1 0) to V_{AC3} only
- ② Perform an AC analysis over the frequency range of interest
- ③ Obtain the following small-signal parameters:

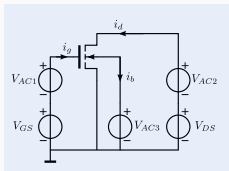
$$g_{mb} = \text{Re} \{ I(V_{AC2}) \}$$

$$C_{bs} = - \frac{\text{Im} \{ I(V_{AC1}) \}}{2\pi f}$$

$$C_{bd} = - \frac{\text{Im} \{ I(V_{AC2}) \}}{2\pi f}$$

Determination of small-signal parameters

Simulation test bench



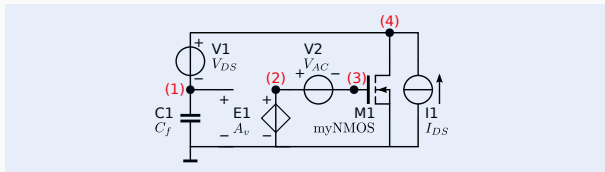
- ① Now add the AC signal (AC 1 0) to V_{AC2} only
- ② Perform an AC analysis over the frequency range of interest
- ③ Finally obtain the following small-signal parameters:

$$g_o = \frac{1}{r_{ds}} = -\operatorname{Re} \{I(V_{AC2})\}$$

$$C_{dg} = -\frac{\operatorname{Im} \{I(V_{AC1})\}}{2\pi f}$$

Determination of cut-off frequency

Simulation test bench



- PSPICE and SIMETRIX parametric analysis with goal functions:
 - $\text{XatNthY}(\langle \text{expr} \rangle, \langle \text{yValue} \rangle, \langle \text{crossingNR} \rangle)$
 - $\text{expr: } \frac{I_D}{I_G} = \frac{I(V_1)}{I(V_2)}$
 - $\text{yValue: } 1$
 - $\text{crossingNR: } 1$